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ORIGINAL ARTICLE

A note on the unsteady torsional sinusoidal flow of fractional viscoelastic fluid in an annular cylinder

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Abstract In this note, the velocity field and the associated shear stress corresponding to the torsional oscillatory flow of a generalized second grade fluid, between two infinite coaxial circular cylinders, are determined by means of Laplace and Hankel transforms. Initially both cylinders and fluid are at rest and then the two cylinders suddenly start torsional oscillations around their common axis with simple harmonic motions having angular frequencies ω_1 and ω_2 . The solutions that have been obtained are presented under integral and series forms in terms of the generalized G and R functions and satisfy the governing differential equation and all imposed initial and boundary conditions. The respective solutions for the motion between the cylinders, when one of them is at rest, can be obtained from our general solutions. Furthermore, the corresponding solutions for the similar flow of ordinary second grade fluid and Newtonian fluid are also obtained as limiting cases of our general solutions. At the end, flows corresponding to the Newtonian, second grade and generalized second grade fluids are shown graphically by plotting velocity profiles.

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1. Introduction

Flows in the neighborhood of spinning or oscillating bodies are of interest to both academic workers and industry. Among them, the flows between oscillating cylinders are some of the most important and interesting problems of motion. As early as 1886, Stokes established an exact solution for the rotational oscillations of an infinite rod immersed in a classical linearly viscous fluid. Casarella and Laura (1969) obtained an exact solution for the motion of the same fluid due to both longitudinal and torsional oscillations of the rod. Later,

Rajagopal (1983) found two simple but elegant solutions for the flow of a second grade fluid induced by the longitudinal and torsional oscillations of an infinite rod. These solutions have been already extended to Oldroyd-B fluids by Rajagopal and Bhatnagar (1995). Others interesting results have been recently obtained by Khan et al. (2005), Fetecau and Fetecau (2006) Mahmood et al. (2009), Vieru et al. (2007), Fetecau et al. (2008), Massoudi and Phuoc (2008), Khan et al. (2009), and Mahmood et al. (2010).

Recently, the fractional calculus has encountered much success in the description of viscoelasticity. Specifically, rheological constitutive equations with fractional derivatives play an important role in the description of the properties of polymer solutions and melts. The starting point of the fractional derivative model of non-Newtonian fluids is usually a classical differential equation which is modified by replacing the time derivative of an integer order by so-called Riemann–Liouville fractional differential operator. This generalization allows us to define precisely non-integer order integrals or derivatives (Podlubny, 1999).

It is important to mention here that a number of research papers in the literature (Fetecau et al., 2008; Massoudi and Phuoc, 2008; Khan et al., 2009; Mahmood et al., 2010) are devoted to the study of the flow of different viscoelastic fluids between two cylinders, when only one of them is oscillating and other is at rest. On the other hand, the exact solutions corresponding to the flow of these fluids between two cylinders, when both of them are oscillating along or around their common axis simultaneously, are very rare in literature. Recently, Mahmood et al. (2009, 2010) have studied the flow of fractional Maxwell and second grade fluids between two cylinders, when both of them are oscillating around, respectively, along their common axis. As far as the knowledge of authors is concerned, in the literature, no attempt has been made to study the flows of fractional second grade fluid due to torsional oscillations of two cylinders. Therefore, in this paper, we are interested into the torsional oscillatory motion of a generalized second grade fluid between two infinite coaxial circular cylinders when both of them are oscillating around their common axis with given constant angular frequencies ω_1 and ω_2 . Velocity field and associated tangential stress of the motion are determined by using Laplace and Hankel transforms and are presented under integral and series forms in terms of the generalized G and R functions. It is worthy to point out that the solutions that have been obtained satisfy the governing differential equation and all imposed initial and boundary conditions as well. The solutions corresponding to the ordinary second grade fluid and those for Newtonian fluid, performing the same motion, are also determined as special cases of our general solutions. Furthermore, the respective solutions for the oscillatory motion between the cylinders, when one of them is at rest, can be obtained from our general solutions.

2. Torsional oscillations between two cylinders

Among the many constitutive assumptions that have been employed to study non-Newtonian fluid behavior, one class that has gained support from both the experimentalists and the theoreticians is that of Rivlin–Ericksen fluids of second grade. The Cauchy stress tensor $\mathbf{T}$ for such fluids is given by

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \quad (1)$$

where $-p$ is the pressure, $\mathbf{I}$ is the unit tensor, μ is the coefficient of viscosity, α_1 and α_2 are the normal stress moduli and $\mathbf{A}_1$ and $\mathbf{A}_2$ are the kinematic tensors defined through

$$\begin{aligned} \mathbf{A}_1 &= \text{grad } \mathbf{v} + (\text{grad } \mathbf{v})^T, \\ \mathbf{A}_2 &= \frac{d\mathbf{A}_1}{dt} + \mathbf{A}_1(\text{grad } \mathbf{v}) + (\text{grad } \mathbf{v})^T \mathbf{A}_1. \end{aligned} \quad (2)$$

In the above relations, $\mathbf{v}$ is the velocity, d/dt denotes the material time derivative and grad the gradient operator. Since the fluid is incompressible, it can undergo only isochoric motions and hence

$$\text{div } \mathbf{v} = \text{tr } \mathbf{A}_1 = 0. \quad (3)$$

If this model is required to be compatible with thermodynamics, then the material moduli must meet the following restrictions:

$$\mu \geq 0, \quad \alpha_1 \geq 0 \quad \text{and} \quad \alpha_1 + \alpha_2 = 0. \quad (4)$$

The sign of the material moduli α_1 and α_2 has been the subject of much controversy. A comprehensive discussion on the restrictions given in (4), as well as a critical review on the fluids of differential type, can be found in the extensive work of Dunn and Rajagopal (1995).

Generally, the constitutive equation of the generalized second grade fluids has the same form as (1), but $\mathbf{A}_2$ is defined by

$$\mathbf{A}_2 = D_t^\beta \mathbf{A}_1 + \mathbf{A}_1(\text{grad } \mathbf{v}) + (\text{grad } \mathbf{v})^T \mathbf{A}_1, \quad (5)$$

where D_t^β is the Riemann–Liouville fractional calculus operator of order β with respect to t defined by

$$D_t^\beta f(t) = \frac{1}{\Gamma(1-\beta)} \frac{d}{dt} \int_0^t (t-\tau)^{-\beta} f(\tau) d\tau, \quad 0 < \beta \leq 1, \quad (6)$$

where $\Gamma(\cdot)$ is the Gamma function. When $\beta = 1$, Eq. (5) may be simplified as (2)₂ while for $\alpha_1 = 0$ the constitutive relationship (1) describes the Rainer–Rivlin viscous fluid.

2.1. Mathematical formulation of the problem and governing equation

Suppose that an incompressible generalized second grade fluid is situated in the annular region between two infinite straight circular cylinders of radii R_1 and $R_2 (> R_1)$ as shown in Fig. 1. At time $t = 0$, the fluid and cylinders are at rest. At time $t = 0^+$, inner and outer cylinders suddenly begin to oscillate around their common axis ($r = 0$) with the velocities $W_1 \sin(\omega_1 t)$ and $W_2 \sin(\omega_2 t)$, where ω_1 is the frequency of velocity of inner cylinder and ω_2 is that of outer cylinder. Owing to the shear, the fluid between the cylinders is gradually moved, its velocity being of the form

$$\mathbf{v} = \mathbf{v}(r, t) = v(r, t) \mathbf{e}_\theta, \quad (7)$$

where $\mathbf{e}_\theta$ is the unit vector along θ -direction. For such flows the constraint of incompressibility is automatically satisfied.

Introducing (7) into the constitutive equation, we find that

$$\tau(r, t) = (\mu + \alpha_1 D_t^\beta) \left(\frac{\partial}{\partial r} - \frac{1}{r} \right) v(r, t), \quad (8)$$

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