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ORIGINAL ARTICLE

Consideration of transient stream/aquifer interaction with the nonlinear Boussinesq equation using HPM

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Abstract The phenomenon of stream–aquifer interaction was investigated via mathematical modeling using the Boussinesq equation. A new approximate solution of the one-dimensional Boussinesq equation is presented for a semi-infinite aquifer when the hydraulic head at the source is an arbitrary function of time. The differential equations were solved using the method of Homotopy Perturbation. The simplicity and accuracy of the approximation are compared with "exact" solution and illustrated numerically and graphically. The results reveal that the HPM is very effective and simple and provides highly accurate solutions for nonlinear differential equations.

1. Introduction

The study of stream–aquifer hydraulics is of great interest as several flow and contaminant problems can be modeled, understood and quantified. It is an important problem in studying of alluvial aquifer. Alluvial valley aquifers are hydraulically connected to their adjacent channels and exchange flow through the streambed (Perkins and Koussis, 1996). The connection causes ground water levels in these systems to fluctuate with respect to the other. Small changes in the stream elevation can cause a large variation in the groundwater elevation in the aquifer. If the stream stage increases over a short period of time, a flow reversal between the channel and aquifer will occur as a result of a change in the hydraulic gradient (Workman et al., 1997). A flood wave is then propagated into the aquifer and increases bank storage. While the stream is returning to normal flows, the bank storage is released. The quantification of the hydraulics of the stream–aquifer in an alluvial valley require a good knowledge of the controlling input hydro-geological parameters, such as hydraulic conductivity, specific yield, recharge, as well as boundary conditions

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(Srivastava et al., 2006). Alluvial valley aquifers pose some interesting boundary conditions. One side of the flow domain is the river source/sink, which is fluctuating. As a result of the alluvial valley formation, the other side of the flow domain is the valley wall, which is a no flow boundary condition. The spread of contaminants in stream–aquifer systems from the river to the aquifer or from the aquifer to the river is also of concern (Serrano et al., 2007).

The hydraulics of the stream–aquifer system could be studied via the solution of the Laplace equation subject to a nonlinear free-surface boundary condition, and time-dependent river boundary conditions. In this way, the groundwater flow in an unconfined aquifer may be approximately modeled by the nonlinear Boussinesq equation, assuming Dupuit's hypothesis of zero resistance to vertical flow is valid, to be a viable alternative to the use of Laplace's equation. With the Boussinesq equation, the vertical coordinate does not exist, and the free-surface boundary condition is not needed (Serrano and Workman, 1998). The result is a simplified model where the effect of time-dependent river boundary conditions can easily be incorporated into the analysis. Solutions of the Boussinesq equation are applied in catchment hydrology and base flow studies as well as agricultural drainage problems and constructed, subsurface wetlands (Lockington et al., 2000).

The governing equation for one-dimensional, lateral, unconfined groundwater flow similar to the Fig. 1 with Dupuit assumptions is the Boussinesq equation (Bear, 1979):

$$\begin{aligned} &\frac{1}{S}\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right) = \frac{\partial h}{\partial t} \quad 0 \leqslant x \leqslant l_x \\ &h(0,t) = H_1(t),\ h(l_x,t) = H_2(t),\ h(x,0) = H_0(t) \end{aligned} \tag{1}$$

where $h(x,t)$ is the hydraulic head (m); K is the aquifer hydraulic conductivity (m/day); S is the aquifer specific yield; $H_1(t)$ and $H_2(t)$ are the time fluctuating heads at the left and right boundaries, respectively; x is the spatial coordinate (m); l_x is the horizontal dimension of the aquifer (m); t is the time coordinate (day); and $H_0(x)$ is the initial head across the aquifer (m).

Until recently, analytical solutions of nonlinear partial differential equations were rare, due to the lack of systematic solution methods. Next section is a very brief review on some analytical methods for this class of equations. One of the most beneficent of these methods is the Homotopy Perturbation Method (HPM). In this paper, we solve the Boussinesq equation by means of HPM and then compare the obtained results with exact solution.

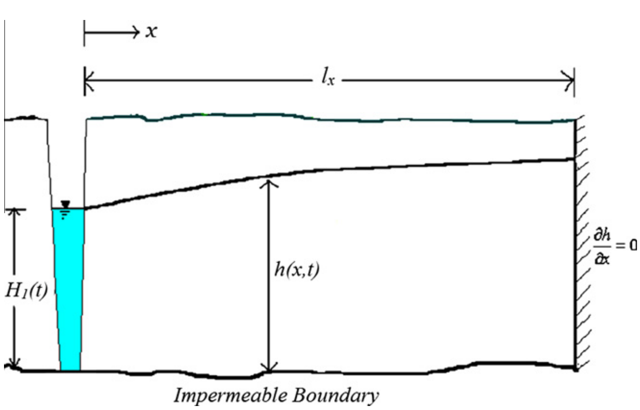

Figure 1 Idealized cross section for the mathematical modeling of transient stream–aquifer interaction.

2. Analysis of the Homotopy Perturbation Method

Nonlinear phenomena play a crucial role in applied mathematics and physics. Although it is very easy for us now to find the solutions of some problems by means of computers, it is still rather difficult to solve nonlinear problems either numerically or theoretically or obtaining an exact solution for these problems. So, it is often more useful to have an approximate closed form solution to describe a nonlinear problem. In recent decades, numerical analysis and the approximate methods have been developed considerably for nonlinear partial equations. More recently, some promising analytical techniques have been proposed, such as Lindstedt–Poincaré (He, 2002b,c), Parameter-Expanding (Shou and He, 2007; Ganji et al., 2009a), Parameterized Perturbation (He, 1999), Harmonic Balance (Telli and Kopmaz, 2006; Gottlieb, 2006), Linearized Perturbation (He, 2003), Energy Balance (He, 2002a; Momeni et al., 2010; Ganji et al., 2009), Variational Approach (Xu, 2008; He, 2004; Ganji et al., 2008), Max–Min (Babazadeh et al., 2010; Ibsen et al., 2010), Exp-Function (Ganji et al., 2009b; Mohyud-Din et al., 2010), Amplitude–Frequency Formulation (Ganji et al., 2010b), Adomian Decomposition (Mirgolbabaei et al., 2010; Wazwaz, 2005), Variational Iteration (Faraz et al., 2011; Babaelahi et al., 2009; Barari et al., 2008a,b; Fouladi et al., 2010), and the Homotopy Perturbation Method (He, 2000, 2005; Ghotbi et al., 2008; Omidvar et al., 2010; Miansari et al., 2010; Ganji et al., 2009c, 2010a).

The Homotopy Perturbation Method is a combination of the classical perturbation and Homotopy technique. To explain this, we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0, \quad r \in \Omega, \tag{2}$$

Subject to boundary condition:

$$B(u, \partial u/\partial n) = 0, \quad r \in \Gamma \tag{3}$$

where A is a general differential operator, B a boundary operator, $f(r)$ is a known analytical function, Γ is the boundary of domain Ω and $\partial u/\partial n$ denotes differentiation along the normal drawn outwards from Ω. The operator A can be divided into two parts: a linear part L and a nonlinear part N. Therefore Eq. (2) can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0, \tag{4}$$

In case that the nonlinear Eq. (2) has no "small parameter", we can construct the following Homotopy:

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p(N(v) - f(r)) = 0, \tag{5}$$

Where,

$$v(r,p) : \Omega \times [0,1] \to R, \tag{6}$$

In Eq. (6), $p \in [0,1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Eq. (5) can be written as a power series in p, as follows:

$$v = v_0 + pv_1 + p^2v_2 + \ldots, \tag{7}$$

Then the best approximation for the solution is:

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \ldots, \tag{8}$$