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ORIGINAL ARTICLE

A comparison of HPM, NDHPM, Picard and Picard–Padé methods for solving Michaelis–Menten equation

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Abstract The fact that physical phenomena are modelled, mostly, by nonlinear differential equations underlines the importance of having reliable methods to solve them. In this work, we present a comparison of homotopy perturbation method (HPM), nonlinearities distribution homotopy perturbation method (NDHPM), Picard, and Picard–Padé methods to solve Michaelis–Menten equation. The results show that NDHPM possesses the smallest average absolute relative error 1.51(−2) of all tested methods, in the range of $r \in [0, 5]$. Also, we introduce the combination of Picard's iterative method and Padé approximants as an alternative to reduce complexity of Picard's solutions and increase accuracy.

1. Introduction

Many important physical phenomena on the engineering and science fields are frequently modelled by nonlinear differential equations. Such equations are often difficult or impossible to solve analytically. Nevertheless, analytical approximate methods to obtain approximate solutions have gained importance in recent years. There are several methods employed to find approximate solutions to nonlinear problems like homotopy perturbation method (HPM) He (2004, 2009, 1999), Biazar and Aminikhah (2009), Biazar and Ghazvini (2009a), Koak et al. (2011), Vázquez-Leal et al. (2012), Vazquez-Leal et al. (2012a), Filobello-Nino et al. (2012b), Khan et al. (2011a), Biazar and Ghazvini (2008, 2009b), Sheikholeslami et al. (2012), Filobello-Nino et al. (2012a), Picard's iterative method Ramos (2009), Szinvelski et al. (2006), Layton and Lenferink (1995), Rach (1987), Bellomo and Sarafyan (1987), Lal and Moffatt (1982), Adomian decomposition method El-Sayed et al. (2010), Li (2009), Ezzati and Shakibi (2011), Safari et al. (2009), Hojjati and Jafari (2008), Abidi and Omrani

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(2010), homotopy analysis method (HAM) Rashidi and Dinarvand (2009), Bataineh et al. (2009), Abbasbandy and Shivanian (2011), Tan and Abbasbandy (2008), Mastroberardino (2011), Abbasbandy (2008), Shukla et al. (2012), Gorder and Vajravelu (2009), Chen and Liu (2008), Qian and Chen (2010), Abbasbandy (2010), Wang (2011), Varitional iteration method He (2012), Turkyilmazoglu (2011), Geng (2011), Altintan and Ugur (2009), Shang and Han (2010), Chen and Wang (2010), Saadati et al. (2009), Odibat and Momani (2009), among others. In this paper, we will present a comparison of nonlinearities distribution homotopy perturbation method (NDHPM) Vazquez-Leal et al. (2012a), Filobello-Nino et al. (2014), homotopy perturbation method, Picard's method and Picard–Padé method to solve Michaelis–Menten equation Golinik (2010, 2011), Gonzalez-Parra et al. (2011). The obtained results show that NDHPM possesses the smallest average absolute relative error $1.51(-2)$ in the range of $r \in [0, 5]$. In addition, we introduce the combination of Picard's iterative method and Padé approximants as an alternative to reduce complexity of Picard's solutions in order to increase accuracy.

This paper is organized as follows. In Section Appendix A, we introduce the basic idea of HPM method. Section 2 presents the basic concept of NDHPM method. We introduce the Picard–Padé coupled method in Section 3. Section 4 presents the approximated solutions of a case study by NDHPM, HPM, Picard and Picard–Padé methods. In Section 5, numerical illustrations are performed and the results discussed. Finally, a brief conclusion is given in Section 6.

2. Distribution of nonlinearities for HPM method (NDHPM)

A recent report Vazquez-Leal et al. (2012a), Filobello-Nino et al. (2014) introduced the NDHPM method, which eases the searching process of solutions for (A.3) and reduces the complexity when solving differential equations. As first step, the homotopy of the form Vazquez-Leal et al. (2012a) is introduced

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p(L(v) + N(v,p) - f(r,p)) = 0, \quad p \in [0,1]. \tag{1}$$

It can be noticed that the homotopy function (1) is essentially the same as (A.4), except for the non-linear operator N and the non-homogeneous function f, which contain the embedded homotopy parameter p. The arbitrary introduction of p within the differential equation is a strategy to redistribute the nonlinearities between the successive iterations of the HPM method, and thus, increase the probabilities of finding the sought solution.

Again, we establish that

$$v = \sum_{i=0}^{\infty} v_i p^i, \tag{2}$$

when $p \to 1$, it turns out that the approximate solution for (A.1) is

$$u = \lim_{p\to 1} v = \sum_{i=0}^{\infty} v_i. \tag{3}$$

3. Picard method and Padé aftertreatment

Given a first order nonlinear differential equation, it can be expressed as

$$y'(r) = L + N + f(r), \tag{4}$$

having initial conditions

$$y(r^*) = K, \tag{5}$$

where L is a linear operator, N is a nonlinear operator, and $f(r)$ is a known function for the independent variable r.

The basic formulation of Picard iterative method is

$$y_{i+1}(r) = K + \int_{r^*}^{r} y_i(v)dv, \tag{6}$$

where the last equation involves n integrals, y_i is the right side of Eq. (4), and r^* is the expansion point.

Usually, the application of Picard's method generates large mathematical expressions difficult to handle. Therefore, we propose the use of a coupling between Picard's method and Padé approximants to generate compact expressions on one hand and increase accuracy Khader (2012) on the other.

3.1. Padé approximants

A rational approximation to $f(r)$ on $[a, b]$ is the quotient of two polynomials $P_N(r)$ and $Q_M(r)$ of degrees N and M, respectively. We use the notation $R_{N,M}(r)$ to denote this quotient. The $R_{N,M}(r)$ Padé Merdan et al. (2011), Baker (1975), Khader (2012), Noor and Mohyud-Din (2009), Bararnia et al. (2012), Raftari and Yildirim (2011), Sangaranarayanan and Rajendran (1997), Nallasamy and Rajendran (1998), Rajendran (2000) approximation to a function $f(r)$ has been given by

$$R_{N,M} = \frac{P_N(r)}{Q_M(r)} \quad \text{for } a \leqslant r \leqslant b. \tag{7}$$

The method of Padé requires that $f(r)$ and its derivatives be continuous at $r = 0$. The polynomials used in (7) are defined as follows

$$P_N(r) = p_0 + p_1 r + p_2 r^2 + \cdots + p_N(r^N), \tag{8}$$

$$Q_M(r) = q_0 + q_1 r + q_2 r^2 + \cdots + q_M(r^M). \tag{9}$$

The polynomials in (8) and (9) are constructed so that $f(r), R_{N,M}(r)$, and their derivatives up to $N + M$ are equal at $r = 0$. For the case when $q_0 = 1$, the approximation is only the Maclaurin expansion for $f(r)$. For a fixed value of $N + M$, the error is the lowest when $P_N(r)$ and $Q_M(r)$ have the same degree or when $P_N(r)$ is one degree higher than $Q_M(r)$.

Notice that the constant coefficients of Q_M are M. This is permissible because 0 and $R_{N,M}(r)$ are not changed when both $P_N(r)$ and $Q_M(r)$ are divided by the same constant. Hence, the rational function $R_{N,M}(r)$ has $N + M + 1$ unknown coefficients. Assuming that $f(r)$ is analytic and has the Maclaurin expansion

$$f(x) = a_0 + a_1 r + a_2 r^2 + \cdots + a_k r^k + \cdots, \tag{10}$$