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ORIGINAL ARTICLE

Solving the interaction of electromagnetic wave with electron by VIM

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Abstract In this paper the interaction of electromagnetic wave with electron is studied by Variational Iteration Method. This phenomenon is very important in physics and one of its application is, generating the High-Order Harmonics from plasma surface. Obtained results are in excellent agreement with experimental results and show the efficiency of applied technique.

1. Introduction

One of the most important applications of differential equations is modeling the phenomena that happen in the nature. But the non-linear part that exists in most of these equations makes it difficult to obtain the exact solution and finding an appropriate method that gives the best approximation is a very big challenge. In recent decades, numerical calculation methods are good means of analyzing these equations. But in the numerical techniques, besides the volume of computational work, stability and convergence should be considered in order to avoid divergent or inappropriate results. So, these techniques cannot be used in a wide class of differential equations and it seems using some analytical techniques such as Homotopy Perturbation Method (HPM) (Rajeev and Kushwaha, 2013; Ebadian and Dastani, 2012; Sheikholeslami et al., 2012a), Adomian Decomposition Method (ADM) (Wazwaz et al., 2013; Gharsseldien and Hemida, 2009; Sheikholeslami et al., 2012a,b, 2013) Variational Iteration Method Using He's Polynomials (VIMHP) (Matinfar and Ghasemi, 2010, Matinfar and Ghasemi, 2013) can end the problems that arise in solving procedures. One of the most important phenomena in nonlinear optics is generating harmonics of the highest possible order. As we know nonlinear optical processes become more efficient at higher laser intensities, but in some cases the best quality of changes in the nature of the nonlinearity of the laser–matter interaction can be seen in certain characteristic intensity regimes (Voitiv and Ullrich, 2001; Voitiv et al., 2002).

Harmonic generation by an intense light wave incident on a plasma-vacuum boundary involves a very complex and collective interaction of the electrons with the electromagnetic field and can be investigated by oscillating mirror model. Oscillating mirror approximation (Voitiv et al., 2005; Dorner et al., 2000; Ullrich et al., 2003; Moshammer et al., 1996) consists of two distinct steps: in the first step the details of the electron spatial distribution are ignored and the collective electronic motion is represented by the motion of some characteristic electronic boundary, e.g., the critical density surface. This sur-

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face represents the oscillating mirror from which the incident light is reflected with the notification of having fixed ions. In the second step, the emission from the moving boundary is calculated, in particular the harmonic spectrum that is generated upon reflection of the incident light.

As we know, the over dense plasma is highly reflective and we have both electric and magnetic fields due to the incident and reflected waves. Therefore, the electrons near the plasma boundary should be driven from both fields. It is in the case that inside the plasma, the electromagnetic fields decay exponentially over a distance given by the skin depth. The motion equation of an electron near the boundary is

$$m\frac{d^2\vec{r}}{dt^2} = -e\vec{E}_1 - e\vec{E} - e\vec{V}\times\vec{B} = \vec{F}_p + \vec{F}_{em}, \tag{1}$$

where E_1 is the longitudinal electric field and resulted from the electron-ion charge separation. The light with the electronic and magnetic field strengths of E and B is acting on the electron with force F_{em}. A qualitative picture of the motion can be obtained by considering the orbit of a single free electron under the action of the electromagnetic wave of frequency w_0, and neglecting restoring force F_p (Corkum, 1993; Brabec, 2008).

This paper is advocated to investigate this phenomenon by the Variational Iteration Method (VIM). The rest of this paper is organized as follows: Section 2 describes the details of the proposed method. Section 3 indicates sufficient conditions for convergence of applied technique. Section 4 explains related partial differential equations which interaction of electromagnetic wave with electron are obtained from and solving procedure. Section 5 shows the simulation results. Finally, conclusions are presented in Section 6.

2. Variational Iteration Method

The Variational Iteration Method, which provides an analytical approximate solution, is applied to various nonlinear problems (Biazar et al., 2010; Ganji et al., 2009; Gholami and Ghambari, 2011; Hassan and Alotaibi, 2010; Khader, 2013; Kafash et al., 2013). In this section, we present a brief description of VIM. This approach can be implemented, in a reliable and efficient way, to handle the following nonlinear differential equation

$$L[u(r)] + N[u(r)] = g(r), \qquad r > 0, \tag{2}$$

where $L = \frac{d^m}{dr^m}, m \in N$, is a linear operator, N is a nonlinear operator and $g(r)$ is the source inhomogeneous term, subject to the initial conditions

$$u^{(k)}(0) = c_k, \quad k = 0, 1, 2, \ldots, m-1. \tag{3}$$

where c_k is a real number. According to the He's Variational Iteration Method, we can construct a correction functional for Eq. (2) as follows:

$$u_{i+1}(r) = u_i(r) + \int_0^r \lambda(\tau)\{Lu_i(\tau) + N\tilde{u}_i(\tau) - g(\tau)\}d\tau, \qquad i \geqslant 0,$$

where $\lambda(\tau)$ is a general Lagrangian multiplier and can be identified optimally via variational theory. As we can see, because of the existence of nonlinear part in Eq. (2), it is not possible to find the optimal value of Lagrange multiplier exactly. But we can find an approximation of that by considering a restriction on nonlinear part that causes this part to be ignored in procedure for calculating $\lambda(\tau)$ and is denoted by $\tilde{u}_i$. The restriction that is mentioned is having restricted variation i.e. $\delta\tilde{u}_i = 0$. Making the above functional stationary with $\delta\tilde{u}_i = 0$,

$$\delta u_{i+1}(r) = \delta u_i(r) + \delta\int_0^r \lambda(\tau)\{Lu_i(\tau) - g(\tau)\}d\tau,$$

yields the following Lagrange multipliers,

$$\begin{cases} \lambda = -1 & \text{for} \quad m = 1, \\ \lambda = \tau - r, & \text{for} \quad m = 2, \end{cases} \tag{4}$$

and in general,

$$\lambda = \frac{(-1)^m}{(m-1)!}(\tau - r)^{(m-1)}, \quad \text{for} \quad m \geqslant 1.$$

The successive approximations $u_i(r), i \geqslant 0$ of the solution $u(r)$ will be readily obtained upon using the obtained Lagrange multiplier and by using selective function u_0 which satisfies initial conditions. In our alternative approach we can select the initial approximation u_0 as

$$u_0 = \sum_{k=0}^{m-1} \frac{c_k}{k!} r^k. \tag{5}$$

Consequently, the exact solution may be obtained as follows

$$u(r) = \lim_{i\to\infty} u_i(r).$$

3. Convergence analysis

In order to study the convergence of the Variational Iteration Method, according to the approach of VIM presented in the previous section, consider the following equation:

$$L[u(r)] + N[u(r)] = g(r). \tag{6}$$

Based on what illustrated above, the optimal value of Lagrange multiplier in general case can be found as:

$$\lambda = \frac{(-1)^m}{(m-1)!}(\tau - r)^{(m-1)}, \quad \text{for} \quad m \geqslant 1.$$

So, we have

$$u_{n+1}(r) = u_n(r) + \int_0^r \frac{(-1)^m}{(m-1)!}(\tau - r)^{(m-1)} \times \{L[u(\tau)] + N[u(\tau)] - g(\tau)\}d\tau.$$

Now, define the operator $A[u]$ as

$$A[u] = \int_0^r \frac{(-1)^m}{(m-1)!}(\tau - r)^{(m-1)}\{L[u(\tau)] + N[u(\tau)] - g(\tau)\}d\tau, \tag{7}$$

and components $v_k, k = 0, 1, 2, \ldots$, as

$$\begin{cases} v_0 = u_0, \\ v_1 = A[v_0], \\ v_2 = A[v_0 + v_1], \\ \vdots \\ v_{k+1} = A[v_0 + v_1 + v_2 + \cdots + v_k]. \end{cases} \tag{8}$$

we have:

$$u(r) = \lim_{k\to\infty} u_k(r) = \sum_{k=0}^{+\infty} v_k. \tag{9}$$