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ORIGINAL ARTICLE

Unsteady viscous flow over a shrinking cylinder

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Abstract The unsteady viscous flow over a shrinking cylinder with mass transfer is studied. Using a similarity transformation, the unsteady Navier–Stokes equations are reduced to nonlinear ordinary differential equations. Numerical technique is used to solve these equations for some values of the parameters involved, namely suction and the unsteadiness parameters. The effects of these parameters on the velocity and the skin friction coefficient are investigated and graphically presented. Results indicate that dual solutions exist for a certain range of suction and unsteadiness parameters.

1. Introduction

The study of fluid flow over a stretching cylinder has attracted the interest of many researchers. It should be mentioned that the boundary layer flow due to a stretching or shrinking surface is a relevant type of flow appearing in many industrial and engineering processes. There are several applications in the engineering processes, for example in polymer and metallurgy industries such as manufacture and extraction of polymer and rubber sheets, melt-spinning, hot rolling, paper production, wire drawing and glass-fiber production, etc. In these situations, the quality of the final product depends to a great extent on the rate of cooling in process and the process of stretching/shrinking (Bachok et al., 2012). Another example of flow toward a shrinking sheet is a rising, shrinking balloon (Wang, 2008). Wang (1988) investigated the steady flow of an incompressible viscous fluid outside a stretching hollow cylinder in an ambient fluid at rest. This problem was then extended by Ishak et al. (2008a) by including the suction and injection effects. It was reported that injection reduces the skin friction as well as the heat transfer rate at the surface while suction acts in the opposite manner.

Wang and Ng (2011) obtained similarity solution for flow due to a stretching cylinder with partial slip condition at the surface. They found that the slip effect significantly decreases the magnitude of the fluid velocity and the shear stress. Later, Wang (2012) solved the problem of a natural convection on a vertical stretching cylinder. The result obtained is an exact similarity solution of the Navier–Stokes equations. In a subsequent paper, Ishak et al. (2008b) have solved numerically the

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problem of magnetohydrodynamic flow and heat transfer over a stretching cylinder. They observed that the heat transfer rate at the surface decreases with increasing values of the magnetic parameter, while the magnitude of the skin friction coefficient increases as the magnetic parameter and the Reynolds number increase.

Recently, Fang et al. (2011) studied the unsteady viscous flow over an expanding stretching cylinder which provides an exact similarity solution to the Navier–Stokes equations. They indicated that the reversal flows exist due to the expansion of the cylinder and the flow field is strongly affected by Reynolds number and the unsteadiness parameter. Later, Fang et al. (2012) reported the numerical solution of the unsteady viscous flow on the outside of an expanding or contracting cylinder. The solution is an exact solution to the unsteady Navier–Stokes equations.

Compared to a stretching cylinder as mentioned above, there is a limited contribution to the problem of an unsteady viscous flow over a shrinking cylinder. The problem of the steady stagnation point flow of a viscous and incompressible fluid over a permeable shrinking circular cylinder was solved numerically by Lok and Pop (2011). They found that up to three solutions exist for a certain range of the shrinking parameter. It is worth mentioning that the flow over a shrinking sheet was considered by Bhattacharyya et al. (2011), Fang et al. (2010), Faraz et al. (2011), Ishak et al. (2010), Lok et al. (2011), Miklavčič and Wang (2006), and Wang (2008), among others. It seems that Miklavčič and Wang (2006) were the first to investigate the flow over a shrinking surface. For this flow configuration, the fluid is stretched toward a slot and the flow is quite different from the stretching case. It has been shown by these authors that mass suction is required generally to maintain the flow over the shrinking sheet. For this new type of shrinking flow, it is essentially a backward flow as discussed by Goldstein (1965). For a backward flow configuration, the fluid loses any memory of the perturbation introduced by the leading edge, say the slot. Therefore, the flow induced by a shrinking surface shows quite distinct physical phenomena from the forward stretching surface.

It is worth mentioning that the unsteady nature of a wide range of fluid flows is of practical importance and has received considerable attention in the past several years. In many applications, the ideal flow environment around the device is nominally steady, but undesirable unsteady effects arise either due to self-induction of the body, or due to fluctuations or nonuniformities in the surrounding fluid. On the other hand, some devices are required to execute time-dependent motion in order to perform their basic functions (McCroskey, 1977). The study of unsteady boundary layers owes its importance to the fact that all boundary layers, which occur in practice are, in a sense, unsteady. Unsteady viscous flows have been studied rather extensively and all the characteristic features of unsteady effects are now more or less familiar to fluid mechanicists. Stewartson (1960), Stuart (1964), Riley (1975,1990), McCroskey (1977), Telionis (1981) and Wang (1989) have concisely reviewed the main ideas and important contributions on the topic. An improved understanding of unsteady fluid flows and the application of this knowledge to new design techniques should provide substantial improvements in performance, reliability, and costs of many fluid dynamic devices.

The aim of the present paper is to investigate the behavior of the unsteady viscous flow over a shrinking cylinder which has not been considered before. The effects of suction and the unsteadiness parameters on the flow field are also investigated. The governing partial differential equations are first reduced to nonlinear ordinary differential equations, before being solved numerically using a shooting method for some values of the governing parameters.

2. Mathematical formulation

Consider the flow of an unsteady, laminar, viscous and incompressible fluid past a permeable infinite cylinder in shrinking motion as shown in Fig. 1. The diameter of the cylinder is assumed as a function of time with unsteady radius $a(t) = a_0\sqrt{1-\beta t}$. Let u and w be the velocity components in the r and z directions, respectively.

For incompressible fluids without body force and based on the axisymmetric flow assumptions, and there is no azimuthal velocity component, the unsteady Navier–Stokes equations governing the flow are (Fang et al., 2011,2012;)

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right) \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) \tag{3}$$

where z and r are the cylindrical polar coordinates measured in the axial and radial directions, respectively, p is the pressure, ν is the kinematic viscosity, β is the constant of expansion/contraction strength, and ρ is the fluid density. We assume that the boundary conditions of these equations are

$$\begin{aligned} &u = \frac{U}{\sqrt{1-\beta t}}, \quad w = -\frac{1}{a_0^2}\frac{4\nu z}{1-\beta t} \quad \text{at} \quad r = a(t) \\ &w = 0 \quad \text{as} \quad r \to \infty \end{aligned} \tag{4}$$

where U (<0) is the constant mass transfer (suction) velocity and a_0 is a positive constant.

Using the similarity variables (Fang et al., 2012)

$$\begin{aligned} &u = -\frac{1}{a_0}\frac{2\nu}{\sqrt{1-\beta t}}\frac{f(\eta)}{\sqrt{\eta}}, \quad w = \frac{1}{a_0^2}\frac{4\nu z}{1-\beta t}f'(\eta), \\ &\eta = \left(\frac{r}{a_0}\right)^2\frac{1}{1-\beta t}, \end{aligned} \tag{5}$$

Eq. (1) is satisfied automatically and since there is no longitudinal pressure gradient, the Navier–Stokes Eq. (3) reduces to the following ordinary differential equation

$$\eta f''' + f'' + ff'' - f'^2 - S(\eta f'' + f') = 0. \tag{6}$$

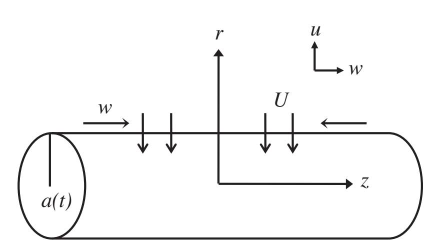

Figure 1 The physical model and coordinate system.