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ORIGINAL ARTICLE

Simultaneous heat and mass transfer in natural convection about an isothermal vertical plate

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Abstract The homotopy analysis method has been used to derive a highly accurate analytic solution for simultaneous natural convection and mass transfer from an isothermal vertical plate. Velocity, temperature, and concentration profiles are presented for a fixed Prandtl number of 0.71 and for Schmidt numbers of 0.5, 5, and 10 and for the buoyancy ratio of 0 (pure mass transfer), 0.5 (simultaneous heat and mass transfer), and 1 (pure heat transfer). The present results corroborate well with the numerical results reported in other research literature on the problem. The auxiliary parameter in the homotopy analysis method is derived by using the averaged residual error concept which significantly reduces the computational time. The use of optimal auxiliary parameter provides a superior control on the convergence and accuracy of the analytic solution.

1. Introduction

The problem of simultaneous heat and mass transfer from an isothermal vertical flat plate is of fundamental importance to the thermal science community. The pioneering contributions to the problem were made by Gebhart and Pera (1971), Mollendorf and Gebhart (1974), Taunton et al. (1970), and Bottemanne (1972a,b). Because of simultaneous heat and mass transfer, the transport process is driven by the interaction of velocity, concentration and thermal boundary layers. The velocity, concentration, and temperature profiles depend on whether the buoyancy forces due to temperature difference and concentration difference aid (aiding flow) or oppose each other (opposing flow). Bottemanne (1972a,b) considered mass transfer due to the injection of water vapor from the plate into the surrounding fluid. He found that for the aiding flow, the heat and mass transfer processes can be treated independent of each other and the results combined to predict the case when the two processes occur simultaneously. This work was examined by Schenk et al. (1976) who noted that Bottemanne's conclusion was valid because the Prandtl number $Pr = 0.71$ and Schmidt number $Sc = 0.94$ used by him were nearly the same. The numerical

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computations of Schenk et al. (1976) led to the conclusion that for $Pr = 0.7$ and $0.6 < Sc < 0.9$, the mutual interaction of heat and mass transfer processes was indeed minimal and the two processes can be treated separately and the two results added. This approach gives results that are in error by about 2%, when compared with the result from the simultaneous analysis. The numerical method used by Schenk et al. (1976) involved an iterative procedure to solve the coupled similarity equations for the stream function, temperature, and concentration.

In the present work, we revisit the problem considered by Bottemanne (1972b) and provide a highly accurate analytical solution of the problem using the homotopy analysis method (HAM). In recent years, the homotopy analysis method (HAM) (Liao, 1999, 2003, 2009; Liao and Tan, 2007), has been successfully applied to many nonlinear problems in science and engineering (Molabahrami and Khani, 2009; Khani et al., 2009a,b,c; Khani and Aziz, 2009; Darvishi and Khani, 2009; Aziz and Khani, 2009). Unlike the perturbation techniques, HAM is independent of any small physical parameters. More importantly, unlike the perturbation and non-perturbation methods, HAM provides a simple way to ensure the convergence of series solution so that one can always get accurate enough approximations even for the strongly nonlinear problems. Furthermore, HAM provides the freedom to choose the so- called auxiliary linear operator so that one can approximate a nonlinear problem more effectively by means of better base functions, as demonstrated by Liao and Tan (2007). The degree of freedom is so large that even the second-order nonlinear two-dimensional Gelfand equation can be solved by means of a 4th-order auxiliary linear operator within the framework of the HAM as shown in Liao and Tan (2007). Especially, by means of the HAM, a few new solutions of some nonlinear problems (Liao, 2005, 2006) have been achieved which otherwise were not solvable by other analytic methods.

2. Formulation of the problem

The physical model and the coordinate system are shown in Fig. 1 where heat and water vapour are transferred simultaneously from the vertical flat plate to the environment. The

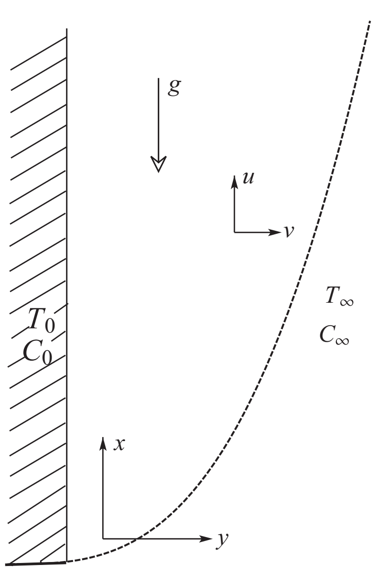

Figure 1 Physical model and coordinate system.

directions of the velocity components u and v are also indicated. The boundary layer equations for this model can be written as (Bottemanne, 1972a; Schenk et al., 1976)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\frac{\rho_\infty - \rho}{\rho} + v\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = a\frac{\partial^2 \theta}{\partial y^2}, \tag{3}$$

$$u\frac{\partial \Omega}{\partial x} + v\frac{\partial \Omega}{\partial y} = D\frac{\partial^2 \Omega}{\partial y^2}. \tag{4}$$

where ρ is the local density, ρ_∞ is the density of the fluid outside the boundary layers, θ is the local temperature difference, a is the thermal diffusivity of the fluid, Ω is the local mass fraction difference, and D is the mass diffusivity. The proper boundary conditions are:

$$\begin{cases} x > 0 & y = 0 & u = v = 0 & \theta = \theta_0 & \Omega = \Omega_0, \\ x > 0 & y = \infty & u = 0 & \theta = 0 & \Omega = 0, \\ x = 0 & y > 0 & u = 0 & \theta = 0 & \Omega = 0. \end{cases} \tag{5}$$

The classical method of solving the system of Eqs. (1)–(5) may be found in most heat and mass transfer textbooks. The same set of equations were considered by Bottemanne (1972a,b). To start the procedure, we have to introduce an expression for the density ρ as a function of the temperature θ and the concentration Ω. This expression is taken from Bottemanne (1972b), who derived it from the assumption that the ideal gas law applies to the air vapour mixture about the vertical wall. If we also introduce the well known stream function substitutions of von Mises and thereupon the similarity transformation of Pohlhausen, we finally obtain (Bottemanne, 1972b):

$$\frac{d^3 f}{d\eta^3} - 2\left(\frac{df}{d\eta}\right)^2 + 3f\frac{d^2 f}{d\eta^2} + \delta\upsilon + (1-\delta)\omega = 0, \tag{6}$$

$$\frac{d^2 \upsilon}{d\eta^2} + 3Prf\frac{d\upsilon}{d\eta} = 0, \tag{7}$$

$$\frac{d^2 \omega}{d\eta^2} + 3Scf\frac{d\omega}{d\eta} = 0, \tag{8}$$

with boundary conditions

$$f(0) = 0 \quad f'(0) = 0 \quad \upsilon(0) = 1 \quad \omega(0) = 1, \tag{9}$$

$$f'(+\infty) = 0 \quad \upsilon(+\infty) = 0 \quad \omega(+\infty) = 0. \tag{10}$$

In this formulation f, υ, and ω represent the reduced stream function, temperature and concentration respectively; the independent variable $\eta = cyx^{-1/4}$, where the constant c depends on the buoyancy forces. The Prandtl and Schmidt numbers have their usual definitions: $Pr = \upsilon/a$ and $Sc = \upsilon/D$. The parameter δ represents essentially the ratio of the thermal buoyancy to the total body force; so is $(1-\delta)$ the ratio of concentration buoyancy to the total effect. For aiding (upward) buoyancy forces δ is necessarily $0 < \delta < 1$. In the next section, we solve the system of non-linear ordinary differential Eqs. (6)–(10) analytically using HAM.

3. HAM solution

In view of the boundary conditions (9) and (10), $f(\eta)$, $\upsilon(\eta)$, and $\omega(\eta)$ can be expressed by the set of base functions of the form