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Desingularized boundary integral equations and their applications in wave dynamics and wave–body interaction problems

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Abstract

Over the past 30 years or so, desingularized boundary integral equations (DBIEs) have been used to study water wave dynamics and body motion dynamics. Within the potential flow modeling, unlike conventional boundary integral methods, a DBIE separates the integration surface and the control (collocation) surface, resulting in a BIE with non-singular kernels. The desingularization allows simpler and faster numerical evaluation of the boundary integrals, and consequently faster numerical solutions. In this paper, derivations of different forms of DBIEs are given and the fundamental aspects and advantages of the DBIEs are reviewed and discussed. Numerical examples of applications of DBIEs in wave dynamics and body motion dynamics are given and the outlook of future development of the desingularized methods is discussed

Keywords: Desingularized boundary integral equations; Potential flows; Wave dynamics; Wave–body interactions.

1. Introduction

Recently, a review paper on desingularized boundary integral equation methods (DBIEMs) and their application in wave hydrodynamics and body dynamics involving water waves and floating bodies was presented at the Prof. R.F. Beck Honoring Symposium on Marine Hydrodynamics of the ASME 2015 34th OMAE Conference in St. John's, Newfoundland, Canada [24]. Due to the limit on the paper numbers of the conference paper, many details could not be included. This paper is an expansion of the OMAE paper providing more information on the DBIEMs and their applications.

For many flow problems involving free surface waves, the flows can be assumed inviscid and irrotational. Subsequently, the flow can be described using a scale function called a velocity potential that is governed by the Laplace equation. With the potential flow assumption, the wave dynamics problem reduces to solving an initial boundary value problem for the velocity potential satisfying proper boundary conditions on the free surface, surfaces of the structures, other rigid surfaces (such as the sea bottom), and a far-field radiation condition.

Solution approaches to most wave problems involve solving a mixed boundary value problem (BVP) for the velocity potential. The BVP can be solved using different methods. Boundary integral equation methods (BIEMs) have been most widely used. A conventional BIEM reformulates the BVP into a boundary integral equation (BIE). The integrals in the BIE involve fundamental singularities distributed over the "integration surface". The strength of the singularities is numerically determined by collocating the BIE on the "control surface". Once the singularity strength is determined, the solution to the BVP can be obtained. In a conventional BIEM, the domain boundary serves as both the "integration surface" and the "control surface". The integrands of the BIE become singular when a point on the "control surface" coincides with a point on the "integration surface".

A so-called desingularized boundary integral equation (DBIE) can be obtained by separating the "integration surface" and the "control surface". The integrands in the integrals in the DBIE are not singular because a point on the "control surface" will never coincide with any point on the "integration surface". The desingularization allows use of

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simpler and faster methods for the numerical evaluation of the panel integrals without degrading the accuracy, resulting in a significant reduction in the computational complexity and time. When the separation of the "integration surface" and the "control surface" are sufficiently far, the integration of the singularity distribution over a panel can be simplified into an isolated singularity, thus further reducing the complexity of the computation. The main advantage of the DBIEM is that it is much easier to program (as compared to the conventional BIEM) and can give a fast solution. A numerical solution method based on a DBIE is referred as desingularized boundary integral method (DBIEM). There can be different types (versions) of DBIEMs, depending on how the DBIE is derived, which will be further described in the later sections in this paper.

The steady flow around a Rankine ovoid that can be found in most fluid mechanics books is probably the best known example of the indirect version of DBIEM. The solution can be obtained by using a source and sink pair combined with a uniform stream to yield a closed stream surface surrounding the two singularities. The closed stream surface is an elongated body, the Rankine ovoid, in a free stream. The distance between the source and the sink, as well as their strength, can be chosen so that the resulting closed streamline surface reassembles the desired Rankine ovoid. The method for the Rankine ovoid flow was extended by von Karman [54] to the steady flow of an arbitrary axisymmetric body in a free stream aligned with the body axis by distributing singularities along the axis inside the body. The strength of the singularity distribution is determined by enforcing the kinematic boundary condition on the body surface.

Webster [57] solved the steady flow past an arbitrary 3-D smooth body using a DBIEM by placing flat triangular panels of sources "submerged' within the body surface with a bilinear distribution of source strength over each panel. The flow solution is constructed as the sum of the uniform incident stream and the flow induced by the source distributions over the triangular panels. The strength of the source distributions is determined by enforcing the kinematic boundary condition at a set of control (collocation) points on the body surface. From the numerical results, Webster concluded that "submergence of the singularity sheet below the surface of the body appears to improve greatly the accuracy, as long as the sheet is not submerged too far". In Webster [57], the panel integrations were done analytically, which require evaluation of transcendental functions.

Kupradze [37] proposed a DBIEM for the exterior Dirichlet problem with an auxiliary control surface outside the problem domain and gave a proof of the uniqueness of a direct version of the DBIE for Dirichlet problems. Heise [32] studied some numerical properties of a DBIE used for plane elastostatic problems. Schultz and Hong [46] used a desingularized complex BIE for two-dimensional potential problems derived from the Cauchy's integral (theorem) and showed the advantages of the desingularization. They also used an overdetermined system combining the real and imaginary parts of the Cauchy's theorem. It was shown that the overdetermined system could exhibit higher-order convergence than the determined system from either the real part or the imaginary part of the Cauchy's integral.

Use of DBIEMs was not popular as compared to singular BIEMs, especially in solving water wave problems. Few applications of DBIEMs used for water wave problems were reported before the early 1980s. Preliminary attempts of using DBIEMs for ship wave problems were reported by Cao [9,10], Mei [41] and Jensen et al. [33]. In solving the steady nonlinear ship wave-making problem, Cao [9,10] used a modified BVP formulation in which the free surface was divided into two zones: the wave zone being the Kelvin wave region bounded by the two $19^\circ 28'$ straight lines starting from a small distance upstream of the ship bow and the non-wave zone being the remaining of the free surface outside the Kelvin wave zone. The nonlinear free surface boundary condition was applied in the deformed free surface in the wave zone and flat rigid horizontal wall condition was applied in the non-wave zone, through which the radiation condition (no waves traveling towards the upstream) was enforced. In solving the modified BVP, Rankine sources were distributed above the free surface and inside the ship hull. The strengths of the sources were determined iteratively by enforcing the boundary conditions at the collocation points on the hull surface and the free surface (both on the deformed free surface in the wave zone and the flat horizontal surface in the non-wave zone). Mei [41] used Webster's "submerged source panel" method Webster [57] to solve the double-body flow which was needed in the Dawson method for calculating the ship waves and the wave-making resistance. Jensen et al. [33] also reported independently the use of a Rankine source distribution above the free surface to solve the nonlinear steady ship wave problems.

Since Longuet-Higgins and Cokelet [40] first introduced the mixed Euler–Lagrange method (ELM) to study two-dimensional fully nonlinear unsteady water waves near breaking, the ELM method extended later to three dimensional problems has become the most popular numerical method for fully nonlinear wave problems in the time domain. The method is a time-marching procedure that requires two major tasks at each time step. In the first task (Euler phase), a BVP is solved for the flow. Then, in the second task (Lagrange phase), the free surface elevation and the velocity potential on it are updated at the next time instant by integrating in time the free surface kinematic condition and dynamic boundary condition. In the time marching approach, most computational time is spent in solving the BVP at each time step. Reducing the computational time in solving the BVP with sufficient accuracy is very critical in simulating wave dynamics for a long duration for practical applications in ships and offshore structures. A research group at the University of Michigan led by Prof. Robert F. Beck started in 1987 to conduct extensive and more systematic investigation on DBIEMs in combination with the Euler-Lagrange time marching approaches to solve fully nonlinear wave problems. During a period of about 15 years, various variations of the DBIEMs and computer algorithms were developed and used by the members of the group (during or after the work at the University of Michigan) to