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Observations and computations of narrow Kelvin ship wakes

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Abstract

Computations of far-field ship waves, based on linear potential flow theory and the Hogner approximation, are reported for monohull ships and catamarans. Specifically, far-field ship waves are computed for six monohull ships at four Froude numbers $F \equiv V/\sqrt{gL} = 0.58$, 0.68, 0.86, 1.58 and for six catamarans with nondimensional hull spacing $s \equiv S/L = 0.25$ at two Froude numbers $F_s \equiv V/\sqrt{gS} = 1$ and 2.5. Here, g is the gravitational acceleration, V and L denote the ship speed and length, and S is the separation distance between the twin hulls of a catamaran. The computations show that, although the amplitudes of the waves created by a ship are strongly influenced by the shape of the ship hull, as well known, the ray angles where the largest waves are found are only *weakly* influenced by the hull shape and indeed are mostly a *kinematic* feature of the flow around a ship hull. An important practical consequence of this flow feature is that the apparent wake angle of general monohull ships or catamarans (with arbitrarily-shaped hulls) can be estimated, without computations, by means of simple analytical relations; these relations, obtained elsewhere via parametric computations, are given here. Moreover, the influence of the two parameters F_s and s that largely determine the ray angles of the dominant waves created by a catamaran is illustrated via computations for three catamarans with hull spacings $s = 0.2$, 0.35, 0.5 at four Froude numbers $F_s = 1$, 1.5, 2, 2.5. These computations confirm that the largest waves created by wide and/or fast catamarans are found at ray angles that only depend on F_s (i.e. that do not depend on the hull spacing s) in agreement with an elementary analysis of lateral interference between the dominant waves created by the bows (or sterns) of the twin hulls of a catamaran. The dominant-waves ray angles predicted by the theory of wave-interference effects for monohull ships and catamarans are also compared with the observations of narrow Kelvin ship wakes reported by Rabaud and Moisy, and found to be consistent with these observations.

Keywords: Kelvin wake; Narrow ship wake; Monohull ship; Catamaran; Wave interference; Bow wave; Stern wave.

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1. Introduction

The far-field waves created by a monohull ship of length L, or a catamaran with two identical hulls of length L separated by a lateral distance S, that advances at constant speed V along a straight path in calm water of large depth are considered. The waves created by a monohull ship or by a catamaran are observed from a Galilean frame of reference attached to the moving ship. The Z axis is vertical and points upward, and the undisturbed free surface is taken as the plane $Z = 0$. The X axis is chosen along the path of the ship, and points toward the ship bow. For a catamaran, the X axis is taken half way between the twin bows of the catamaran. Nondimensional coordinates

$$(x, y, z) \equiv (X, Y, Z)/L \tag{1}$$

are defined.

A famous analysis of this basic problem has been given by Kelvin in 1887. Kelvin's classical far-field asymptotic analysis is based on a one-point wavemaker model of a ship, linear potential flow theory, and the method of stationary phase. The one-point wavemaker model is a major approximation, and indeed is the most restrictive assumption of Kelvin's analysis of ship waves. Specifically, within the context of linear potential flow theory considered by Kelvin, as well as here, the flow around a ship hull is represented via a distribution of sources over the ship hull surface. Interference among the waves cre-

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ated by the sources distributed over the hull surface therefore occurs. However, these wave-interference effects are ignored in the one-point wavemaker model considered by Kelvin.

The main result of Kelvin's analysis, given in every textbook on water waves and ship hydrodynamics, is that a ship creates transverse and divergent waves that exist inside a wedge with half angle

$$\psi^K \approx 19^\circ 28' \tag{2}$$

Moreover, Kelvin showed that the wave pattern formed by the transverse and divergent waves only depends on the ship speed V. Thus, Kelvin's wave pattern does not depend on the length L of a ship, or on the lateral separation distance S between the twin hulls of a catamaran, and therefore does not depend on the Froude numbers

$$F \equiv V/\sqrt{gL} \quad F_s \equiv V/\sqrt{gS} \equiv F/\sqrt{s} \tag{3}$$

where g is the acceleration of gravity and

$$s \equiv S/L \tag{4}$$

denotes the nondimensional hull spacing of the catamaran. Kelvin's wave pattern is also independent of the hull shape, and indeed is the same for every ship (low-speed monohull ship, high-speed catamaran, submarine, hovercraft). Specifically, the Kelvin wave pattern only depends on the coordinates $(X, Y)g/V^2$. This classical result is a consequence of Kelvin's highly-simplified model of a ship as a one-point wavemaker.

Numerous experimental observations and numerical computations (including computations reported further on in this study) show that the largest waves created by common displacement ships in fact are found in the vicinity of the cusp lines $\psi = \pm\psi^K$ of the Kelvin wave pattern, in agreement with Kelvin's analysis. However, at high Froude numbers $F_K \le F$ with $F_K \approx 0.6$ for monohull ships, it has long been observed, notably by Taylor [24], Baker [2], Munk [14], Brown [1], Reed [22], Fang [8], Rabaud [23], that the largest waves created by a ship are found along ray angles that are inside the cusp lines $\psi = \pm\psi^K$ of the Kelvin wake; i.e. the largest waves are found at an 'apparent wake angle' that is smaller than ψ^K at high Froude numbers $F_K \le F$. In particular, the 37 observations of ship wakes reported by Rabaud [23] for the wide range of Froude numbers $0.1 < F < 1.7$ include 12 observations for $0.6 < F$. These high-speed observations are consistently and significantly smaller than the Kelvin cusp angle $\psi^K \approx 19^\circ 28'$ and are as small as 7°. The Rabaud–Moisy observations of narrow ship wakes are considered further on.

Several theoretical explanations of the seemingly unexpected observations of 'narrow ship wakes' at high Froude numbers have been offered in the literature. In particular, a number of studies have shown that several effects not considered by Kelvin can significantly affect the far-field waves created by a ship. In particular, the influence of nonlinearities, ambient waves, wind, shear current, surface tension, and water depth is considered by Brown [1], Fang [8], Mei [15], Zhu [25], Pethiyagoda [21], Ellingsen [7], Moisy [16]. However, the fact that effects not considered in Kelvin's classical analysis can significantly modify the wake of a ship does not imply that the observations of narrow ship wakes reported in the literature are actually due to these extraneous effects. In fact, most of the foregoing studies of ship wakes are only remotely related to Kelvin's practical purpose of explaining the wave patterns created by common ships that advance at constant speed in calm water of large depth, and in fact do not explain observed narrow ship wakes. In particular, the foregoing studies do not seek to refine Kelvin's fundamental model of a ship as a one-point wavemaker.

Numerical computations, within the context of linear potential flow theory, of the far-field waves due to Gaussian distributions of pressure at the free surface by Darmon [6], Dias [5], Benzaquen [4], Moisy [17] or distributions of sources over ship hull surfaces by Barnell [3], Zhang [26], He [9,10] show that, at high Froude numbers, the largest waves created by surface-distributions of pressure or sources are found along ray angles that are located inside the cusp lines $\psi = \pm\psi^K$of the classical Kelvin wake. This numerical finding is consistent with the elementary analysis of constructive and destructive interference between the divergent waves created by a two-point wavemaker — specifically, a point source and a point sink located at the bow and the stern of a monohull ship, or two point sources (or sinks) located at the bows (or sterns) of the twin hulls of a catamaran — considered by Noblesse (2014), and He [11] for deep water and by Zhu [29,30] for shallow water.

Thus, the computations of waves due to surface distributions of pressure or sources reported by Darmon [6], Dias [5], Benzaquen [4], Moisy [17], Barnell [3], Zhang [26], He [9,10] as well as the analysis of interference effects for a two-point wavemaker — an elementary approximation to a distribution of sources over a ship hull surface via a point source and a point sink for a monohull ship, or two point sources for a catamaran — given by Noblesse (2014), He [11], Zhu [29,30] suggest that the observations of narrow ship wakes reported in the literature most likely correspond to the rays where the largest divergent waves created by a high-speed ship are found. Indeed, Noblesse [20], and He [11] contend that narrow ship wakes observed for high-speed ships are merely the unsurprising consequence of longitudinal interference between the divergent waves created by sources and sinks distributed over the bow and stern regions of a ship hull surface, and lateral interference between the divergent waves created by sources (or sinks) distributed over the port and starboard sides of a hull surface or the twin hulls of a catamaran. Wave interference — a main feature of the linear potential flow theory of ship waves — is then an extremely simple, indeed nearly trivial, explanation of the observations of narrow ship wakes reported in the literature.

Within the context of the linear potential flow theory considered here, the flow around a ship hull can be represented via a distribution of sources (and sinks) over the surface of the ship hull. A practical and realistic method for determining the ray angles that correspond to the largest waves created by a ship (for general ship hulls, including multihulls, and/or distributions of pressure at the free surface) is given by Zhang [26]. This method is based on the numerical de-