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A fatigue damage *meso*-model for fiber-reinforced composites with stress ratio effect

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ABSTRACT

This work presents a fatigue damage *meso*-model for fiber-reinforced plastic composites, in which the effect of stress ratios on the off-axis fatigue behavior is taken into account. The non-dimensional effective stress concept is introduced in the continuum damage mechanics method. Damage growths and fatigue failure are studied along axial, transverse and shear directions at *meso*-scale level. The proposed model is validated through numerical simulations that describe the *meso* fatigue damage accumulation and the fatigue life for off-axis unidirectional fiber-reinforced plastic composite laminates of arbitrary fiber orientation under different stress ratios. It is shown that the fatigue damage behavior and fatigue life for off-axis unidirectional glass/epoxy and carbon/epoxy composite laminates are adequately described by the proposed fatigue model over the range of different stress ratios.

1. Introduction

Fiber-reinforced composites are widely used in aerospace, marine, automotive and advanced engineering applications in recent years, due to their high-quality mechanical properties. However, these structures always suffer cyclic fatigue loadings during service life, such as aircraft wings, helicopter blades, wind turbine blades and so on [1]. As a consequence, one important issue during the design of these composite structures is the fatigue damage assessment: the strength and durability of the composite structural components must take into account the typical damage phenomena occurring under in-service loading. The fatigue behavior of fiber-reinforced composites is quite different from the one of metals [2], due to their anisotropy and heterogeneity characteristics, and the multi-scale nature of the damage processes and non-linear damage evolution during loading [3]. Therefore, it is important to understand the mechanisms associated to fatigue damage and to predict the long-term fatigue strength and life for fiber-reinforced composites under complex cyclic fatigue loading.

The fatigue damage failure process of fiber-reinforced composites involves a number of different failure mechanisms and interactive coupling effects. The different types of damage include fiber fracture, matrix cracking, matrix crazing, fiber buckling, fiber–matrix interface failure, delamination among composite plies and the effect of shear-induced diffuse damage on transverse cracks in fiber-reinforced composites, which has been already investigated through experimental [4] and theoretical methods [5], respectively. In addition, the fatigue performance of composites is also affected by the constituents of composite system, reinforcement structure, lay-up sequence, residual stress due to manufacturing process [6] and stress ratios [7–9] from external loading conditions. In order to simulate the fatigue damage behavior and to predict fatigue life of fiber-reinforced composites, in recent years several methodologies that implement progressive failure analysis and appropriate constitutive models with damage accumulation laws have been developed. In open literature, fatigue progressive damage models have been extensively established from macro to microscopic scales by means of theoretical analysis methods, finite element solutions and experiments [10–17]. Montesano et al. [18] have established a damage mechanics based model that takes into account local multiaxial stresses as well as variable amplitude cyclic loading. The numerical results from that model showed the capability of that approach to predict the evolution of the damage and the degradation of the material properties in a triaxially braided carbon fiber polymer matrix component. Krüger and Rolfes [19] have presented a new layer-based fatigue damage model (FDM) for laminated multidirectional laminates in general states of plane stress. The stiffness and strength degradation were simulated using a Finite Element (FEM) analysis, and the stress redistributions and sequence effects were also analyzed. Eliopoulos and Philippidis [20] developed an anisotropic non-linear constitutive model implementing progressive damage concepts to predict the residual strength/stiffness and life of composite laminates subjected to multiaxial variable amplitude cyclic loading. In-plane mechanical properties of

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the material were fully characterized at the ply level while static or fatigue strength of any multidirectional stacking sequence can be predicted. Paepegem and Degrieck [21] established a phenomenological residual stiffness model to predict the stiffness degradation and possible permanent strains in fiber-reinforced polymers under in-plane fatigue loading. The stress-strain-damage relationships and the damage growth rate equations were developed and explained thoroughly. Montesano and Singh [22] have developed a multi-scale damage model combining synergistic damage mechanics with an energy-based damage evolution framework to predict the evolution of sub-critical matrix cracks in different plies under multiaxial loading, the ply crack density evolution and the laminate stiffness degradation. Quaresimin et al. [23] investigated the very early stages of the damage evolution under a uniaxial cyclic tensile loading by testing [45/-45/0]s glass/epoxy specimens. In that work the first event observed for the damage initiation was multiple micro-cracks in the interfiber region of the 45° ply, with a specific inclination with respect to the fibers. However, all the studies cited above mainly focus on the evaluation of the fatigue damage behavior, little attempt has been made to interpret the fatigue damage propagation and the effect of complicated loading mode such as stress ratio on fatigue damage growth, as well as the fatigue damage mechanisms for fiber-reinforced composites at *meso*-scale.

Continuum damage mechanics (CDM) is a mathematical and experimental description of the damage accumulation and growth due to changes of the material microstructure. On the basis of CDM theory, Pierre Ladevèze and his group established *meso*-scale damage models to describe the strength deterioration of composites under static loading. It is assumed that the behavior of any stratified structure can be described through two families of basic damageable constituents: the elementary layer and the interlaminar interface, and damage is considered uniform through the thickness of individual layers of composites [24–27]. In these models two damage mechanisms are introduced. The first is related to the diffuse intralaminar damage associated with the fiber/matrix debonding in the ply and with small transverse cracks in the matrix. The second damage mechanism is associated with diffuse interlaminar damage linked to the formation of micro-voids in the matrix of the interlaminar interface, resulting in a reduced stiffness of the interlaminar interface with no visible delamination (Fig. 1 [27]). Therefore, the diffuse damage at the elementary ply scale can be modelled by a stiffness decline of the material along the axial, transverse and shear directions.

It is essential to extend the *meso*-scale damage model associated to static loading to complicated cyclic fatigue configuration. Also, it is quite important to investigate the fatigue damage behaviors and to develop new fatigue prediction methodologies for fiber-reinforced composites at *meso*-scale levels.

In this paper we aim to establish a new fatigue damage meso-model in which the CDM theory is applied with the use of damage variables at the meso-scale of elementary plies and stress ratios to account for the complex fatigue loading history. The model is able to determine the fatigue damage growth at meso-scale and to predict the fatigue life of unidirectional composite laminates with arbitrary fiber orientation under different stress ratios. In this approach the progressive growth of diffuse damage is evaluated by establishing three groups of damage growth rate equations (along the axial, transverse and shear directions) according to continuum damage mechanics. We also introduce a non-dimensional effective stress [8,9] to build a new fatigue diffuse damage meso-model that considers the effects of the fiber orientation and the stress ratios on the off-axis fatigue behavior of unidirectional fiber-reinforced composite laminates. We then evaluate the validity of the proposed fatigue damage meso-model using data from the principal damage variables occurring in tension-tension cyclic loading under high-low stress levels and different stress ratios by GFRP and CFRP unidirectional composite laminates under different stress ratios with constant amplitude and frequency conditions [7,8]. The results from the model are therefore discussed and show the viability of the proposed approach to predict on and off-axis fatigue damage propagation in composites.

1.1. Fatigue damage meso-model

The present fatigue damage *meso*-scale model for unidirectional plies is developed within the framework of the thermodynamics in irreversible phenomena. Under the assumption of plane stresses and small perturbations, the strain energy of the ply can be written in the following form:

$$W_D = \frac{1}{2}\left[\frac{\langle\sigma_{11}\rangle_+^2}{E_{11}^0(1-D_{11})} + \frac{\langle\sigma_{11}\rangle_-^2}{E_{11}^0} - 2\frac{\upsilon_{12}^0}{E_{11}^0}\sigma_{11}\sigma_{22} + \frac{\langle\sigma_{22}\rangle_+^2}{E_{22}^0(1-D_{22})} + \frac{\langle\sigma_{22}\rangle_-^2}{E_{22}^0} + \frac{\tau_{12}^2}{G_{12}^0(1-D_{12})}\right] \quad (1)$$

Where E_{11}^0, E_{22}^0 and G_{12}^0 represents the initials stiffness of fiber, transverse and shear direction in plane, respectively. $\langle\rangle_+$ is defined as the positive part and $\langle\rangle_-$ as the negative parts. Consequently, when $\sigma_{22} \leq 0$, micro-cracks are closed and no noticeable damage occurs. Three damage indicators, which are constant through the thickness, pertain to the following mechanisms: Fiber breakage D_{11} (along the axial direction), matrix micro-cracking D_{22} (along the transverse direction) and deterioration of the fiber-matrix bonds D_{12} (along the shear direction).

From this potential, thermodynamic forces associated with the tension and shear internal variables $D_{ij}(i,j=1,2$ and $i<j)$ are defined:

$$Y_{ij} = \frac{\partial W_D}{\partial D_{ij}} = \frac{\sigma_{ij}^2}{2E_{ij}^0(1-D_{ij})^2} \quad (2)$$

The damage growth rates dD/dN correspond to the damage kinetics and are expressed as a function of the thermodynamic forces Y_{ij}, which are also connected to the applied stress σ_{ij}. Therefore, a typical damage growth equation for a continuum fatigue damage variable$D$ can be represented as:

$$\frac{dD_{ij}}{dN} = \mathbf{f}(D_{ij}, \sigma_{ij}, R, N, p) \quad (3)$$

Where $\mathbf{f}$ defines a fatigue damage function, the parameters σ_{ij}, R, N and p denote applied maximum stress, stress ratio, number of fatigue cycles and a history dependent parameter, respectively.

1.2. The effect of stress ratios

Under realistic service conditions most structural components made from multidirectional composite laminates are subjected to complex

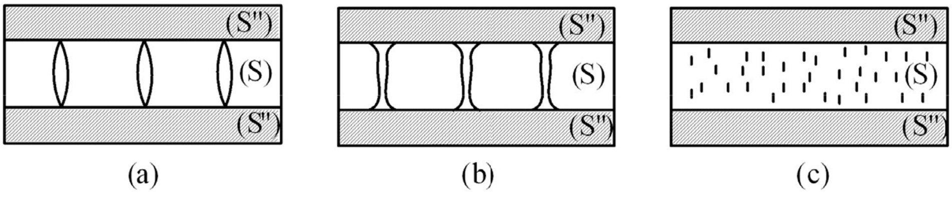

Fig. 1. Mechanisms of degradation on *meso*-scale: (a) transverse matrix microcracking; (b) local delamination; (c) diffuse damage [27].