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Size-dependent buckling and postbuckling behavior of piezoelectric cylindrical nanoshells subjected to compression and electrical load

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ABSTRACT

This paper studies the nonlinear buckling and postbuckling characteristics of piezoelectric cylindrical nanoshells subjected to an axial compressive mechanical load and an electrical load in the presence of surface free energy effects. The electrical field is applied along the transverse direction. A size-dependent shell model is adopted based on the Gurtin–Murdoch elasticity theory and von Karman geometrical nonlinearity. To satisfy the balance conditions on the surfaces of the nanoshell, a linear variation is considered for the normal stress of the bulk through the thickness. A boundary layer theory is employed including surface energy effects in conjunction with the effects of nonlinear prebuckling deformation, large deflections in the postbuckling regime and initial geometrical imperfections. Afterwards, a two-stepped singular perturbation technique is employed to obtain the size-dependent critical buckling load and the associated postbuckling equilibrium path for alternative electric loadings. It is found that the surface free energy and electrical load can cause an increase or decrease on the critical buckling load and the associated postbuckling strength of a nanoshell depending on the sign of surface properties and applied voltage. These anticipations are the same for the both perfect and imperfect piezoelectric nanoshells.

1. Introduction

In recent years, piezoelectric materials with direct and converse effects [1–8] play an important role for several applications in adaptive piezoelectric nanoelectromechanical systems (NEMS). As a result, investigation on the electromechanical characteristics of piezoelectric structures at nanoscale is crucial for NEMS design. Masmanidis et al. [9] and Sadek et al. [10] predicted the vibrational response of piezoelectric MEMS made of GaAs nanofilm and GaAs nanowire, respectively. They controlled the resonance frequency of piezoelectric nanostructures using bias voltage. Chen et al. [11] performed a theoretical study on the buckling and dynamic stability of a piezoelectric viscoelastic nanobeam subjected to van der Waals forces. Liang et al. [12] developed a theoretical model to investigate the surface energy effects on the postbuckling behavior of piezoelectric nanowires. Fang et al. [13] proposed an analytical model using electro-elastic surface/interface model for nanoscale structures to study the dynamic electromechanical response of a multilayered piezoelectric nanocylinder subjected to electro-elastic waves. Recently, Wang et al. [14] analyzed the free and forced vibrations of piezoelectric circular nanoplates considering the effects of surface free energy and nonlocality.

In order to successfully design and develop nanodevices and nanostructures, it is important to consider all essential characteristics of their mechanical behavior at this submicron size. As the classical continuum models have not the capability to consider size-effects in the analysis of nanostructures, their applicability in nanoscale is controversial. Hence, the modification of continuum mechanics to accommodate the size dependency of nanostructures is a topic of major interest. In last decade, several non-classical continuum elasticity models have been proposed in nanomechanics due to their accuracy and computational efficiency. These non-classical continuum models can predict numerical results close to those achieved by molecular dynamics models [15–25].

The surface free energy effect is one of the significant molecular effects which can be easily observed at the atomic scale, and this has been clearly indicated and explained [26,27]. Because of dissimilar environmental conditions, atoms at a free surface need different equilibrium requirements in comparison with the atoms in the bulk material. This difference causes excess surface energy as a superficial energy term

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since a surface can be interpreted as a layer to which certain energy is attached [28]. For nanostructures, the surface free energy effect may be significant due to their high surface area to volume ratio.

Gurtin and Murdoch [29,30] developed a theoretical framework based on the continuum mechanics including surface free energy effects. In their model, the surface of structure is regarded as a mathematical layer with zero thickness adhered to the underlying bulk material. The Gurtin-Murdoch elasticity theory has been applied to many studies to predict the mechanical behavior of nanostructures [31–44]. In recent years, Kiani [45] developed a model based on surface elasticity theory to study the surface effects on the free transverse vibration and instability of current-carrying nanowires immersed in a longitudinal magnetic field. Gao et al. [46] considered the surface stress effects in the analysis of nanowire buckling on elastomeric substrates. Cheng and Chen [47] presented a theoretical study of the resonance frequency and buckling load of nanoplates with higher-order surface stress model. Sahmani et al. [48] predicted the surface free energy effects on the postbuckling characteristic of cylindrical nanoshells subjected to axial compression and in different temperatures. Sahmani et al. [49] studied the free vibration characteristics of postbuckled functionally graded third-order shear deformable nanobeams using surface elasticity theory. Zhang et al. [50] investigated the transverse vibration of an axially compressed nanowire embedded in elastic medium by implementing the high-order surface stress model into the Bernoulli-Euler beam theory. Raghu et al. [51] presented analytical solutions for laminated composite plates using a nonlocal third-order shear deformable theory considering the surface stress effect. Zhang and Meguid [52] examined a modified continuum model of fluid-conveying nanobeams by incorporating the surface elasticity.

Due to the importance of instability phenomenon in nanoscale systems in electronics and biomedical applications, the objective of the current study is to predict the nonlinear axial buckling and postbuckling characteristics of piezoelectric cylindrical nanoshells in the presence of surface free energy effects. It is assumed that the nanoshells are subjected to an axial mechanical compressive load combined with a transverse electrical field. Gurtin-Murdoch elasticity theory is implemented into the classical shell theory to develop an efficient size-dependent shell model incorporating surface free energy effects. The non-classical governing differential equations with transverse displacement and stress function as independent variables are deduced to a boundary layer problem, which includes simultaneously the effects of surface free energy, nonlinear prebuckling deformation, large postbuckling deflections and initial geometric imperfections. Subsequently, a two-stepped singular perturbation technique is put to use in order to obtain the size-dependent critical buckling load and the associated postbuckling equilibrium path of piezoelectric nanoshells subjected to a combined electromechanical load.

2. Preliminaries

In Fig. 1, a cylindrical nanoshell made of PZT-5H piezoelectric material with the length L, thickness h, and mid-surface radius R is shown which is subjected to axial compressive load combined with electric field. The nanoshell includes a bulk part and two additional thin surface layers (inner and outer layers).For the bulk part, the material properties are Young's modulus E and Poisson's ratio ν. The two surface layers are assumed to have surface elasticity modulus of E_s, Poisson's ratio ν_s and the surface residual tension τ_s. According to a curvilinear coordinate system with its origin located on the middle surface of nanoshell, coordinates of a typical point in the axial, circumferential and radial directions are denoted by x, y and z, respectively. Now, in accordance with the classical shell theory, the displacement field can be expressed as

$$u_x(x,y,z)=u(x,y)\text{-}z\frac{\partial w(x,y)}{\partial x} \tag{1a}$$

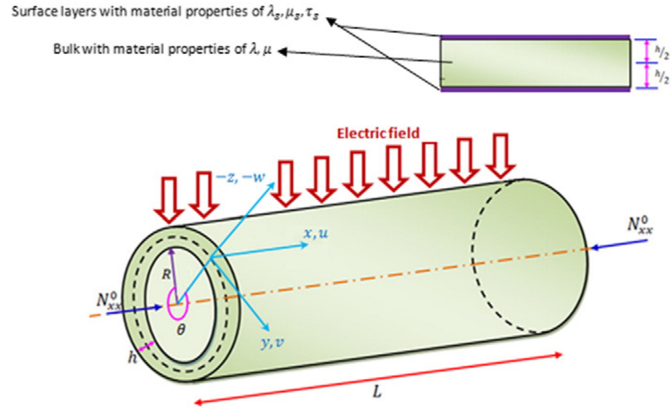

Fig. 1. Schematic view of a piezoelectric cylindrical nanoshell and its surface layers, subjected to axial compression and electric field.

$$u_y(x,y,z)=v(x,y)\text{-}z\frac{\partial w(x,y)}{\partial y} \tag{1b}$$

$$u_z(x,y,z) = w(x,y) + w^*(x,y) \tag{1c}$$

in which u, v and w denote the middle surface displacements along x, y and z axis, respectively, and w^* represents the initial geometric imperfection in nanoshell.

On the basis of the von Karman–Donnell-type kinematics of nonlinearity [53], based on which it is assumed that the thickness of the shell h, is remarkably small in comparison with its radius of curvature R, the kinematical strain–displacement relationships for a cylindrical nanoshell subjected to electric field in thickness direction (E_z) can be expressed as follow

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon^M_{xx}\\ \varepsilon^M_{yy}\\ \gamma^M_{xy}\end{Bmatrix}+\begin{Bmatrix}\varepsilon^E_{xx}\\ \varepsilon^E_{yy}\\ \gamma^E_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon^0_{xx}\\ \varepsilon^0_{yy}\\ \gamma^0_{xy}\end{Bmatrix}+z\begin{Bmatrix}\kappa_{xx}\\ \kappa_{yy}\\ \kappa_{xy}\end{Bmatrix}+\begin{Bmatrix}\frac{d_{31}\mathcal{V}}{h}\\ \frac{d_{31}\mathcal{V}}{h}\\ 0\end{Bmatrix}$$
$$=\begin{Bmatrix}\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w^*}{\partial x}\\ \frac{\partial v}{\partial y}-\frac{w+w^*}{R}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial w^*}{\partial y}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+\frac{\partial w}{\partial y}\frac{\partial w}{\partial x}\end{Bmatrix}-z\begin{Bmatrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\frac{\partial^2 w}{\partial x\partial y}\end{Bmatrix}+\begin{Bmatrix}\frac{d_{31}\mathcal{V}}{h}\\ \frac{d_{31}\mathcal{V}}{h}\\ 0\end{Bmatrix} \tag{2}$$

where $\varepsilon^M_{xx},\varepsilon^M_{xx},\gamma^M_{xy}$ and $\varepsilon^E_{xx},\varepsilon^E_{xx},\gamma^E_{xy}$ represent the mechanical and electrical strain components, respectively. Also, $\varepsilon^0_{xx},\varepsilon^0_{xx},\gamma^0_{xy}$ stand for the strain components of the middle surface, and $\kappa_{xx},\kappa_{yy},\kappa_{xy}$ denote the curvature components of nanoshell, d_{31} and $\mathcal{V}=E_z h$ are the piezoelectric strain constant and applied voltage across the shell thickness, respectively.

Then the constitutive relations can be given as

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}=\begin{bmatrix}\lambda+2\mu & \lambda & 0\\ \lambda & \lambda+2\mu & 0\\ 0 & 0 & \mu\end{bmatrix}\begin{Bmatrix}\varepsilon^M_{xx}\\ \varepsilon^M_{yy}\\ \gamma^M_{xy}\end{Bmatrix}\text{-}\begin{bmatrix}\lambda+2\mu & \lambda & 0\\ \lambda & \lambda+2\mu & 0\\ 0 & 0 & \mu\end{bmatrix}$$
$$\times\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\frac{d_{31}\mathcal{V}}{h}\\ \frac{d_{31}\mathcal{V}}{h}\end{Bmatrix} \tag{3}$$

in which $\lambda=E\nu/(1-\nu^2)$, $\mu=E/(2(1+\nu))$ are Lame's constants.

Gurtin–Murdoch elasticity theory facilitates considering surface energy effects in the conventional continuum approach. In relation with the atomic features of nanostructures, there are always interactions