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Lateral-torsional buckling behaviour of long-span laminated glass beams: Analytical, experimental and numerical study

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ABSTRACT

This paper presents results of analytical, experimental and numerical studies on the lateral-torsional buckling (LTB) behaviour of long-span laminated glass beams. The analytical study was mainly focused (i) on the assessment of existing expressions for the determination of the effective flexural and torsional stiffness of 3-layer laminated glass beams and (ii) on the determination of the buckling resistance and post-buckling behaviour of long-span laminated glass beams taking into account the influence of changes in the thickness of the glass panes and of the viscoelastic properties of the interlayers. The experimental study comprised a flexural test of a simply supported and unbraced 8.20 m long PVB 3-layer laminated glass fin. Finally, three-dimensional numerical models were also developed in order to simulate the experiments and validate the analytical results. The results obtained show that the analytical formulae and the numerical tools available are able to accurately predict the LTB behaviour of long-span 3-layer laminated beams. The results also draw the attention to the importance of adequately considering the influence of possible thickness reductions on the glass panes and of temperature and loading time effects on the shear behaviour of PVB interlayers.

1. Introduction

Over the last decades architects and engineers have strived to maximize the transparency of building façades through the use of glazed solutions. Their design efforts have ultimately led to all-glass façades with structural glass fins: vertical glass beams perpendicular to the façade plane, which are being increasingly used to transfer horizontal loads to other structural elements. The new production possibilities [1], namely (i) the increase of the glass pane lengths, and (ii) the introduction of interlayers with improved structural performance, are providing longer and more robust glass fins; indeed, it is now possible to cover a two floor height façade with a single element, without any type of connection.

One of the main features of glass fins is their high slenderness, *i.e.* the ratio between total length (L) and height (b), which makes them highly susceptible to the lateral-torsional buckling (LTB) phenomenon. During the last years, several authors have experimentally addressed this issue [2–5], conducting comprehensive experimental campaigns where the influence of the following parameters was analysed: total length, slenderness, initial imperfections, glass type, interlayer material, glass or interlayer thickness, temperature and load duration. In addition, most of the authors have also developed analytical and numerical studies, having reported accurate predictions of the LTB behaviour of 2-layer laminated glass beams. Although providing relevant results, the above mentioned experimental campaigns were limited in terms of the span of the laminated glass beams. In fact, the vast majority of tests were conducted in small-to-intermediate scale specimens and, according to the best of the authors' knowledge, no experiments were reported for beams longer than 3 m.

Previous investigations have also addressed the analytical methods available for the design of those structural glass members to the LTB phenomenon [6–10]. In most studies, the design approach developed for steel structures has been successfully adapted to equivalent monolithic laminated glass beams. The basic procedure relies on the principle of transforming the several layers of the glass fin (glass panes and polymeric interlayers) into a structurally equivalent monolithic element [2, 7,11–13]. The works performed so far can be distinguished based on the approach used to determinate the equivalent bending ($EI_{y,eff}$) and torsional ($GJ_{t,eff}$) stiffnesses of the cross-section. Bedon et al. [9] and Machado-e-Costa [10] recently presented an extensive review of the available formulations, showing that such approaches provide accurate estimates of the LTB behaviour of 2-layer laminated glass fins. The main analytical studies on this field ultimately led to the publication of new design guidelines [14,15] and standards [16,17] that are a result of the main developments achieved so far. Each guideline or regulation presents and adopts a given calculation approach, and the formulae available in the various documents can differ significantly among each other. Therefore, a unified design approach applicable to the most frequent design situations is still missing. Additionally, with the exception

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of the Italian standard [17], there are no expressions available for the design of laminated glass beams with more than 3 glass panes. Furthermore, due to the lack of experiments on long-span beams or on laminated glass beams with more than 3 layers, those design equations have not been duly validated at full-scale.

In addition to experimental and analytical studies, several authors have also developed numerical (finite element, FE) models [4,7] in order to simulate the LTB behaviour of laminated glass beams. Due to the complexity of the problem (*i.e.*, the heterogeneous section or the viscoelastic behaviour of interlayers), numerical modelling is a convenient tool which allows to obtain accurate estimates of the buckling resistance and post-buckling behaviour of laminated glass members. This approach proved to be successful in several previous studies (*e.g.*, [2,4,10,11]); however, as for the experiments, it was only used for small-to- intermediate-scale laminated glass beams.

This paper presents further experimental and numerical investigations about the LTB behaviour of laminated glass beams and assesses the accuracy of analytical formulae. The beams studied herein are similar to the glass fins used in the façade of the *Champalimaud Centre for the Unknown*, in Lisbon. The main distinctive characteristic of these 3-layer glass fins is their dimensions (8.20 m long × 0.60 m high × 0.048 thick), much larger than those used in the previous studies mentioned above. The main goal of the work presented here was threefold: (i) to experimentally investigate the flexural and LTB behaviour of these full-scale beams, including their linear, buckling and post-buckling responses; (ii) to assess the accuracy of the analytical expressions developed by Machado-e-Costa [10] for the design of 3-layer laminated glass beams; and (iii) to investigate the accuracy and (practical) applicability of conventional numerical FE models when applied to such large beams.

2. Geometrical and mechanical properties of the analysed beam

2.1. Beam geometry

The laminated beam analysed in this paper has 3 layers of fully tempered glass with polished edges and 2 PVB interlayers. The nominal geometry of each glass pane is 8200 mm of length, 600 mm of height and 15 mm of thickness. The nominal thickness of the interlayer films is 1.52 mm.

The thickness of the laminated beam, as well as that of the individual components, has remarkable influence on both torsional and bending stiffness in the minor axis, thus it also has significant impact on the LTB behaviour. Furthermore, according to EN 572-8 standard [18], glass panels 15 mm thick may present differences in thickness up to ± 0.5 mm. In what concerns the PVB interlayer, according to Callewaert et al. [19], these elements present a very precise thickness prior to the lamination process. However, differences up to 9% were detected after lamination.

The measured average height was $b_{real} = 601.54$ mm (*vs.* 600 mm) and the mean thickness was $t_{total,real} = 48.26$ mm (*vs.* 48.04 mm). After lamination, with exception of the extremity sections, it is extremely hard to measure the actual thickness of each layer. Nevertheless, the authors were able to conclude that the glass panes used in the beam could not present the nominal thickness of 15 mm; indeed, based on local measurements, the geometry of the beam most likely included 14.50 mm thick glass panels and 2.28 mm thick PVB interlayers.

2.2. Mechanical properties

The mechanical properties considered in the analytical and numerical models are average values taken from the literature, namely from the recent *Guidance for European Structural Design of Glass Components* [14].

For the glass panes, the following properties were considered (soda-lime-silica glass): Young's modulus (E) of approximately 70 GPa (which has been reported to present low scatter), Poisson's ratio of 0.23, and (characteristic) bending strength of 120 MPa (fully tempered glass).

PVB is a thermoplastic polymeric material that presents viscoelastic behaviour in the range of service temperatures for building applications (-20 °C to $+80$ °C). The material is highly susceptible to creep or relaxation phenomena, with mechanical properties being highly dependent on load duration and temperature [20]. The formulation and application of viscoelastic material models (function of time and temperature) is complex and generally of difficult implementation. In most common situations, the structural response of laminated glass for a specific action and environmental condition can be easily and approximately determined by running *quasi-static* analysis considering the general behaviour of the interlayer for a relevant temperature range [9,21]. In the present study, the mechanical properties of PVB (elastic and shear moduli) for load durations commonly considered in the design of laminated glass elements were taken from the study of van Duser et al. [22], obtained from DMA[1] tests at 20 °C (Table 1). Table 1 also presents the mechanical properties of PVB for a temperature of 23 °C, which was the average temperature of the experimental test. It can be seen that a small temperature increase (of only 3 °C) causes remarkable reductions in the mechanical properties of PVB. Finally, according to Callewaert [20], PVB presents tensile strength higher than 20 MPa, Poisson's ratio of 0.49, ultimate axial deformation higher than 250%, and glass-transition temperature of about 20 °C.

3. Analytical study

The analysis of the elastic LTB critical load and of the post-buckling behaviour of laminated glass elements summarized in this section follows the classic theory of structural stability, widely applied to steel members. The main difference between laminated glass and steel members relies on the formulation used to compute the bending stiffness (EI_y) and the torsional stiffness (GJ_t) of the cross-sections. In the case of laminated glass members, these parameters must be computed taking into account the contribution of the two constituent materials, the glass panes and the polymer interlayer (affecting the level of interaction at the interfaces). The prevalent analytical method is based on replacing the stiffness parameters for a homogeneous (monolithic) section (EI_y and GJ_t) by equivalent stiffnesses ($EI_{y,eff}$ and $GJ_{t,eff}$, respectively) corresponding to the laminated glass member.

This section describes the analytical study about the LTB behaviour of the full-scale laminated glass beam. It first describes the method for computing the above mentioned equivalent stiffnesses. Next, the critical LTB load and the post-buckling path are calculated. The final part of the section summarizes the Southwell method, which is applied in Section 4 to estimate the experimental critical load.

3.1. Flexural and torsional stiffness

The equivalent flexural and torsional stiffness can be determined based on several analytical approaches. The ones more frequently used are based on theories originally developed for (i) sandwich structures [2,10], and (ii) composite beams with partial interaction [7]. Several analytical expressions have been published in the literature, for both flexural [3,12,23–25] and torsional stiffness [2,26]. However, most of the works are limited to 2-layer laminated glass beams and the formulae available for multi-layer beams have been shown to be inaccurate for the entire range of mechanical properties of interlayer materials [10,27]. Machado-e-Costa [10] recently published an extensive review and assessment of the accuracy of several different analytical approaches available in literature. The author clarified some

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[1] These curves are obtained from dynamic mechanical analysis (DMA), applying the Williams-Landel-Ferry (WLF) theory; based on small-scale tests carried out for a range of temperatures and load frequencies, it is possible to predict the long-term behaviour of the material.