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Design of lattice structures with controlled anisotropy

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ABSTRACT

Recent advances in additive manufacturing make it possible to fabricate periodic lattice structures with complex configurations. However, a proper design strategy to achieve lattice structures with controlled anisotropy is still unavailable. There is an urgent need to fill this knowledge gap in order to develop mechanical metamaterials with prescribed properties. Here we propose two different methodologies to design lattice structures with controlled anisotropy. As examples, we created two new families of lattice structures with isotropic elasticity and cubic symmetric geometry. The findings of this work provide simple and effective strategies for exploring lightweight metamaterials with desired mechanical properties.

1. Introduction

Cellular structures are widely adopted as biological organisms in nature through generations of natural selection, e.g., wood and bones [9], sponge and diatom [2,23]. Cellular materials including honeycomb and foam-like structures have high strength-to-weight ratios and other superior properties [9]. Thus artificial cellular materials were developed and fabricated in the past decades on a large scale for a broad range of applications such as energy absorption [6], energy storage [16], structural components [28], catalyst carrier and biomaterials [14,15]. A majority of man-made cellular materials are periodic structures due to their stable mechanical properties and ease for mass production. Various periodic structures have been designed in the past to accommodate the increasing demands of applications requiring specific mechanical properties. The increasing advancement of various additive manufacturing technologies makes it possible to fabricate complex structures which cannot be processed by conventional technologies [7,8,10,19]. The challenge is therefore finding the optimal design of structures. A typical case is the design of bone implants. The stiffness of the artificial bone is anisotropic and has to be delicately designed as porous structures to match the neighbouring natural bones and avoid the stress shielding, which may cause significant malfunction after long service. Analyses on natural bone show that the spatial distribution of Young's modulus is smooth without sharp increase or decrease in certain directions. However, most existing periodic bone implants focused on simply reducing the stiffness [1,4,5,11,12,20,22,24,29,33] with very few seeking for structures with isotropic or controlled anisotropic elasticity [1,4,11,33]. Besides, most designs of bone implants were not truss-like lattice and had complex internal structures since they were obtained through topological optimization techniques [4,31,33].

Lattice structures are structurally simple and widely used not only as biomaterials, but also as other components in civil, aerospace and mechanical engineering. Studies on design and manufacturing lattice structures are becoming increasingly important due to the attractiveness of ultra-stiff and ultra-strong metamaterials with exceptional properties [18,25,34]. Design of these structures with directionally controlled mechanical properties is of critical importance in various applications. However, systematic and effective methodologies for designing such lattice structures with controlled anisotropy are still unavailable. A conventional method to create a nearly isotropic lattice structure is to select proper representative unit cells from 3D architecture libraries and make modifications on them to achieve goals [1,5]. This procedure usually involves large amount of work on finite element modelling to evaluate the stiffness of structures in various loading directions through trial-and-error methods, which is tedious and time-consuming. In this work, we propose methodologies from a new perspective which can effectively and efficiently build lattice structures with controlled anisotropy. To achieve this, we first solve the problem of characterization and evaluation of direction-dependent Young's modulus spatial distribution of lattice structures. Previously the Young's modulus spatial surface was used to study the anisotropy of monocrystallines [17,21]. Similarly, it is possible to expand the method to represent the spatial distribution of Young's modulus for lattice structures, if the effective Young's modulus of these structures could be derived. Fortunately, the homogenization theory [26,27] makes this possible since it can accurately approximate the non-continuum periodic composites to continuum ones. Therefore, by combining the homogenization method and the 3D representation of Young's modulus, it becomes easy and straightforward to assess and

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analyse the representative unit cells. Built upon this, we find ways to design novel lattice structures with controlled anisotropy. These design methodologies will be useful for designing bone implants with desired mechanical properties and for creating the next generation ultra-stiff and ultra-strong metamaterials.

2. Methods and calculation

In this work, a simple and straightforward numerical homogenization technique proposed by Steven [26,27] was used to obtain the effective stiffness matrix of the non-continuum, periodic lattice structures. In Hooke's law, $\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$, unknown constants of the forth-order stiffness matrix **C** can be reduced from 81 to 21 due to the symmetry in orthogonal system. In numerical realization [26,27,32], each time one strain component was set to unit whereas the rest five as zero, e.g., Eq. (1).

$$\text{Input}: \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \text{output}: \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{Bmatrix} = \begin{Bmatrix} C_{11} \\ C_{21} \\ C_{31} \\ C_{41} \\ C_{51} \\ C_{61} \end{Bmatrix} \tag{1}$$

In this approach, the unit strain was expressed as a prescribed displacement on the boundary so that the corresponding stresses could be determined from reaction forces in finite element analyses. Therefore, six finite element analyses were used to determine all the components of the stiffness matrix. Two types of boundary conditions were involved in terms of the strain type, i.e., normal strain and shear strain [27]. Take a normal strain ε_x (ε_{11}) for instance, the boundary conditions were defined by

$$\begin{aligned} &\Delta l_x|_{x=l_x} = 0.001 l_x \\ &\Delta l_x|_{x=0} = \Delta l_y|_{y=l_y} = \Delta l_y|_{y=0} = \Delta l_z|_{z=l_z} = \Delta l_z|_{z=0} = 0 \end{aligned} \tag{2}$$

which means the displacement in x axis is $0.001 l_x$ when $x = l_x$, i.e., $\varepsilon_x = 0.001$ and the displacements in all other directions are zeros. The boundary conditions for shear strain cases were defined different. For example, in the case of shear strain γ_{xy} (ε_{12}), the boundary conditions were

$$\begin{aligned} &\Delta l_x|_{z=l_x} = 0.0005 l_z, \Delta l_z|_{x=lz} = 0.0005 l_x \\ &\Delta l_z|_{x=0} = \Delta l_y|_{y=l_y} = \Delta l_y|_{y=0} = \Delta l_z|_{z=l_z} = \Delta l_x|_{z=0} = 0. \end{aligned} \tag{3}$$

Although the boundary conditions were defined only from mathematical viewpoint, this approach provides highly accurate results for predicting the macroscopic mechanical properties of materials and structures [26,27]. The meshing size was sufficiently small so that the influence of the bending moment could be negligible. The whole homogenization procedure was coded in Fortran language with the finite element analysis of each numerical test conducted in ABAQUS.

Tridimensional orientation-dependent polar plots of Young's modulus surface are commonly used to represent the anisotropy of monocrystallines [17,21]. Similar method may also be extended to characterize the non-continuum lattice structures after homogenization treatment of the compliance (or stiffness) matrix. In this work, the Young's modulus values at any direction are obtained through a successive procedure used in classical books [17,21], including calculation of the direction cosine, transformation and rotation of stress tensor in different coordinate systems. The matrix of direction cosine [λ] (3×3 matrix) is predetermined when rotating unitary orthogonal coordinate system. Anisotropic materials abide by the Hooke's law $\{\sigma\} = [C]\{\varepsilon\}$, where $\{\sigma\} = [\sigma_{11}\ \sigma_{22}\ \sigma_{33}\ \sigma_{23}\ \sigma_{31}\ \sigma_{12}]^T$ and $\{\varepsilon\} = [\varepsilon_{11}\ \varepsilon_{22}\ \varepsilon_{33}\ \varepsilon_{23}\ \varepsilon_{31}\ \varepsilon_{12}]^T$. After transformation in the orthogonal coordinate systems, the new stiffness matrix becomes $[C'] = [T][C][T]^T$. The transformation matrix is dependent on the matrix of the direction cosine. Through this operation, the Young's modulus surface can be plotted in a 3D space at any direction defined by the direction cosine (different angles). The calculation was coded in Matlab to produce all the 3D Young's modulus surfaces in this work.

In this work, the stiffness matrix obtained through homogenization was firstly assessed to check the anisotropy. For isotropic structures, there are only two independent elastic constants, C_{11} and C_{12}. In numerical analysis, the anisotropy of the structure can be measured by

$$A = 2C_{44}/(C_{11} - C_{12}). \tag{4}$$

If A is close to unity, the structure could be treated as isotropic.

3. Results and discussion

3.1. Anisotropy of lattice structures

Unlike continuum metal materials, which are macroscopically isotropic due to the random distribution of crystalline grains, lattice structures are commonly elastically anisotropic with evident weak directions relating to the rod arrangement. Using numerical homogenization method proposed by Steven [26,27], we evaluated the effective stiffness matrix of a variety of reported representative unit cells (base units). Plotting the Young's modulus surface in an orthogonal coordinate system similar to the method for analysing the elastic anisotropy of monocrystallines [17,21], the strong and weak directions can be clearly

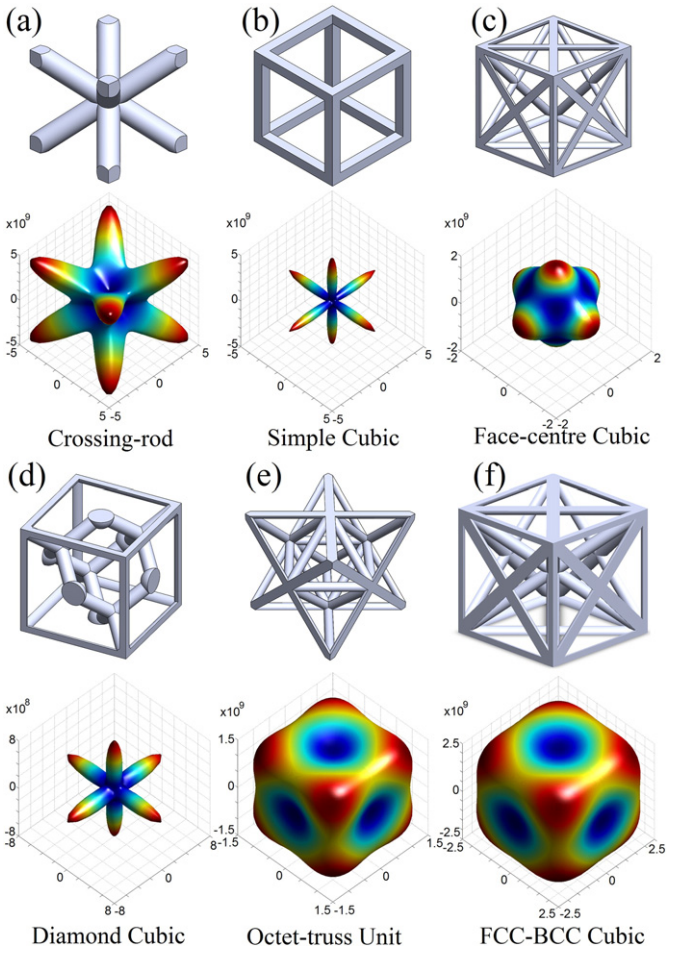

Fig. 1. Architectures of typical representative units and the corresponding 3D spatial representations of effective Young's modulus surfaces: (a) crossing-rod unit, (b) simple cubic unit, (c) face-centre cubic unit, (d) diamond cubic unit, (e) octet-truss unit, and (f) a combined unit of face-centred and body-centred units (FCC–BCC).