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Formulation of a representative plastic strain and representative plastic strain rate by using a conical indentation on a rigid visco-plastic material

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ABSTRACT

The indentation test consists in pressing an indenter on the surface of a tested material and in measuring continually the load F in function of the displacement h of the indenter. In order to analyze the indentation curve $F(h)$ and extract material properties, one method among others uses the notion of the representative strain. The first aim of this paper is to investigate a new concept of a representative plastic strain as well as a new concept of a representative plastic strain rate. The second aim is to couple the effect of work hardening and strain rate to define the corresponding representative plastic strain and the corresponding representative plastic strain rate. It is shown that there are respectively two representative plastic strains and a plastic strain rate: the former obtained using the hardness of indentation and the latter obtained using the loading curve. Then, it is shown that the values of the representative strain and of the strain rate depend on the material behavior and its constitutive parameters.

1. Introduction

Mechanical characterization became an important industrial challenge because of the increase of specific materials used. The instrumented indentation test (Fig. 1) has been developed during the last decades and appears to be a good alternative to the classic mechanical tests to determine the mechanical properties of studied materials [1–4]. The indentation test is a local and quasi-non-destructive test which allows the characterization of the local and global mechanical properties when classical tests cannot be used, such as in the case of a small available volume of material or in the case of a thin layer.

However, evaluating the intrinsic mechanical properties of the materials from indentation data remains a major challenge because the strain field around the indenter is heterogeneous. Then, some approaches have been proposed in order to define a '*representative strain*' depending on the geometrical characteristic of the indenter and on the loading conditions [5–17]. In the case of a self-similar indentation of an elastic–plastic material, e.g. with a conical indenter, the geometrical similarity leads to a representative strain which does not depend on the indentation penetration degree [18]. As a consequence, by using a dimensional analysis, it is shown that during a conical indentation test both the contact hardness H Eq. (1) and the loading curvature C_L Eq. (2) are constant [8,14,18,19]:

$$H = \frac{F}{\pi a_C^2} \tag{1}$$

$$C_L = \frac{F}{h^2} \tag{2}$$

where F is the applied load, a_c is the contact radius and h is the penetration depth of the indenter into the material. This is one of the reasons explaining why one indentation curve obtained from a conical indenter cannot lead to a unique solution in the determination of the material's mechanical properties [19]. For each set of identified parameters, which gives the same indentation loading curves, the identified stress–strain curves cross each other at a single point that corresponds to the representative strain and a representative stress, thus one point in the hardening curve can be assessed. Moreover using two kinds of indentation data, the Hardness H and the curvature C_L, two separate representative strains can be assessed. They can both be used in order to identify the hardening parameters of an elastic plastic material.

One the one hand, Atkins and Tabor [5] was the first to introduce the notion of the representative strain ε_{RP} obtained from

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the hardness estimation. He stated that the flow stress σ_R is related to hardness H by a confined factor C_F defined by the following expression:

$$H = C_F \sigma_R \tag{3}$$

It has been shown that the hardness H of a work-hardening material increases with the angle θ of the indenter. It has also been shown that it is possible to determine the stress–strain curve of copper and mild steel metals using several angles of the cone. In his experimental study, Tabor has suggested, for a Vickers indenter, that $C_F = 3$ and $\varepsilon_{RP} = 0.08$. Using an algebraic approach similar to that used by Tabor, Chaudri [7] has proposed $\varepsilon_{RP} = 0.20$ and $C_F = 3.1$. The two authors have proposed very close values of C_F, however the proposed values of ε_{RP} are very different [5,7,9].

On the other hand, many authors [8,10,11] have shown that during a conical indentation test of an elastic plastic material the curvature C_L in Eq. (2) remains constant. According to the *Π-theorem* [18], a function of k variables can be reduced on $k-p$ dimensionless variables, where p is the number of fundamental units, the loading curvature C_L can also be related to σ_R with Eq. (4) which leads to the determination of another representative strain [8]:

$$C_L = \sigma_R \Pi_1 \left(\frac{E^*}{\sigma_R} \right) \tag{4}$$

where Π_1 is a dimensionless function. This is the reason why the first purpose of this paper is to show that, according to the considered indentation data, the value of the representative strain is different. Moreover, this work will explain the reason of these differences.

Contrary to elastic plastic materials which were widely investigated, time dependent materials have not been investigated thoroughly. However as for the elastic plastic material, a representative strain rate $\dot{\varepsilon}_R$ was proposed for conical indentation of a power law creep solids [20,21], in the aim to apply the same philosophy for a polymer [22–25]. According to a dimensional analysis, using a constant ratio $\frac{\dot{h}}{h}$, the geometrical similarity also occurs and both H and C_L are constant [18]. The representative strain rate $\dot{\varepsilon}_R$ is thus defined as proportional to the $\frac{\dot{h}}{h}$ ratio according to every author, but each gives a different formulation of this representative strain rate. For a Berkovich indenter, most of authors [26–28] have proposed that the representative strain rate is defined by:

$$\dot{\varepsilon}_R = \frac{1}{h}\frac{dh}{dt} = \frac{\dot{h}}{h} \tag{5}$$

Based on this previous work, Bucaille et al. [29] has shown that the representative strain rate increases with θ and he has proposed the following relationship:

$$\dot{\varepsilon}_R = 0.6 \; cotan \; \theta \frac{\dot{h}}{h} \tag{6}$$

where θ is the half-angle of a conical indenter. Later, Kermouche et al. [30] pointed out the influence of the strain rate sensitivity on the representative strain rate and stated that this latter can be approximated by:

$$\dot{\varepsilon}_R \approx 0.44 \; \exp \left(\frac{0.2}{m} \right) \; cotan \; \theta \frac{\dot{h}}{h} \tag{7}$$

Regarding Eqs. (5)–(7), different expressions of the representative strain rate, using the hardness H, are proposed. However, the calculation of the hardness remains another problem due to the experimental measurement of the contact area for these material. Then, an alternative methodology is using the curvature C_L of indentation for the calculation of the representative strain rate.

Firstly the behavior of a rigid plastic material under a conical indentation in order to define the two representative plastic strains for both H and C_L has been investigated. Secondly, a power law creep material has been considered in order to study the influence of the strain rate parameter m for both H and C_L and to suggest an explanation for the different expression of the representative strain rate. Finally, the hardening and the sensitivity to the strain rate have been combined in order to define a representative plastic strain and a representative plastic strain rate for a rigid–viscoplastic material.

2. Representative plastic strain and representative plastic strain rate

2.1. Studied materials and finite element model

In this section, the concept of representative strain for a rigid plastic material is studied, as defined by Eq. (8). The concept of representative strain rate is also studied by considering, for the equivalent stress σ, a simple power law function defined by Eq. (9).

$$\sigma = K\varepsilon_P^n \tag{8}$$

$$\sigma = K\dot{\varepsilon}_P^m \tag{9}$$

A numerical analysis was realized using the commercial finite element code ABAQUS in order to investigate the concepts of the representative strain and representative strain rate. A convergence investigation was performed in order to determine the optimal mesh. Moreover, the mesh near the contact zone was refined to increase the accuracy of the FE model. This latter had 6996 elements with 7158 nodes (Fig. 2). The indenter was considered as

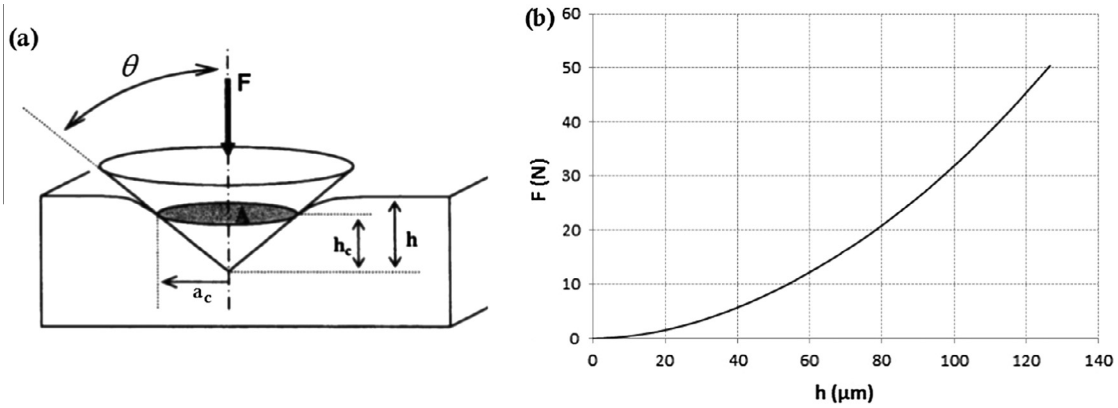

Fig. 1. Illustration of an instrumented indentation test (a) conical indenter where h is the penetration depth, a_c is the contact radius and h_c is the contact depth and (b) loading curve obtained by using a conical indenter.