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Reliability analysis for cementless hip prosthesis using a new optimized formulation of yield stress against elasticity modulus relationship

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ABSTRACT

Using classical design optimization methods for implant-bone studies does not completely guarantee a safety and satisfactory performance, due in part to the randomness of bone properties and loading. Here, the material properties of the different bone layers are considered as uncertain parameters. So their corresponding yield stress values will not be deterministic, that leads to integrate variable limitations into the optimization process. Here there is a strong need to find a reliable mathematical relationship between yield stress and material properties of the different bone layers. In this work, a new optimized formulation for yield stress against elasticity modulus relationship is first developed. This model is based on some experimental results. A validation of the proposed formulation is next carried out to show its accuracy for both bone layers (cortical and cancellous). A probabilistic sensitivity analysis is then carried out to show the role of each input parameter with respect to the limit state function. The new optimized formulation is next integrated into a reliability analysis problem in order to assess the reliability level of the stem–bone study where we deal with variable boundary limitations. An illustrative application is considered as a bi-dimensional example (contains only two variables) in order to present the results in an illustrative 2D space. Finally, a multi-variable problem considering several daily loading cases on a hip prosthesis shows the applicability of the proposed strategy.

1. Introduction

Traditional deterministic design methods have accounted for uncertainties through empirical safety factors. The designer does not take into account uncertainties concerning materials, geometry and loading. A number of uncertainties are encountered during the design of osteo-articular systems. These uncertainties are resulted from the variability of applied loads and materials properties, in addition to that resulting from the design modeling. They can be grouped in three main categories, namely irreducible, reducible and statistical uncertainties [1]. In the best case scenario in the design of structural systems, uncertainties can be reduced or minimized but they cannot be completely eliminated. Thus, all parameters of interest in an engineering design can be considered as random variables [2]. To select these parameters, a sensitivity analysis can be carried out. Some researchers have developed computational tools using a set of random variables in context of probabilistic analysis and durability studies [3,4]. However, predicting the mechanical behavior of bone is also a challenging task for the current research as experimental tests on fresh human bones which are often limited by the difficulty in obtaining specimens and the complexity of testing protocols. Although the local properties of bone changes continuously location by location two main different types of tissue may be distinguished. The outer layer limited by the periosteum is characterized by dense and compact bone which has the highest mechanical properties, it is the cortical bone. The inner part, or spongy bone, is characterized by a higher level of porosity, lower mechanical properties, higher vascularization and a higher . Here, three kinds of ability to absorb energy before fracture; it is the cancellous (or trabecular) bone. Our objective is to assess the reliability level of an optimized formulation relating the elasticity modulus and the yield stress. We first develop a new formulation for the elasticity modulus and yield stress relationship. The developed formulation is next integrated into the reliability analysis and applied on the stem design implanted in a proximal femurvariables can be considered:

1. Design variables x_i: the design variables are deterministic variables defined in order to optimize the system. They represent control parameters of the mechanical system (e.g., dimensions, materials, loads) and of the probabilistic model (e.g., mean values and/or standard-deviations of random variables).

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2. Random variables y_i: the uncertainties are modeled by stochastic physical variables affecting the failure scenario. These variables can represent geometrical dimensions, material characteristics or applied external loading. The knowledge of these variables is not, at best, more than statistical information and it can be admitted as a representation in the form of random variables. The random physical variables represent the structural uncertainties, which are identified by probabilistic distributions.
3. Normalized variables u_i: they represent the transformation of the random variables from the physical space to a normalized one according to certain probabilistic distribution laws.

The material of this paper is organized as follows: some objectives concerning reliability analysis are first presented in Section 2. A review of the previous formulations of material properties, especially, Young's modulus and yield stress against density relationship, are presented in Section 3. We begin Section 4 with our generalized formulation of the yield stress against Young's modulus relationship in Section 4.1. Using some experimental results, the generalized formulation is next developed to find our optimized constants of proportionality in Section 4.2. A numerical validation of the proposed formulation for cortical and cancellous experimental results is carried out in Section 4.3. Two numerical stem–bone examples considering several daily loading cases are presented in Section 5. The first numerical example of a bi-dimensional variable case is considered as an illustrative 2D space modeling and the second one is a multi-dimensional variable case to show applicability of the reliability integration using the proposed formulation and Section 6 concludes the paper.

2. Reliability analysis

The notion of reliability is very old. Ancient civilizations constructed huge buildings and mechanisms and many of these structures still exist, i.e., they have proven to be very reliable designs. However, the cost of construction of these structures was tremendous. Nowadays, the two main objectives in the design of structural systems are to design systems that have satisfactory reliability and are as inexpensive as possible [5]. In this section, some basic concepts are first presented for design under uncertainty and for reliability assessment methods.

2.1. Design under uncertainty

There is no way to make a perfectly safe design. Ignoring uncertainty and using safety factors usually leads to designs with inconsistent reliability levels. Three types of uncertainties can be considered [1]:

1. Irreducible uncertainty: irreducible (or Inherent) uncertainty is due to the inherent randomness in physical phenomena and processes. It arises during the description of a physical process and still exists even if unlimited data is available.
2. Reducible uncertainty: reducible (or model) uncertainty may happen due to the use of imperfect models to predict outcomes of an action. It results from the simplification of modeling a true physical process and can be minimized by using more sophisticated model.
3. Statistical uncertainty: it is due to the lack of data for modeling uncertainty. Or, it is related to the fitting of a parametric distribution and this uncertainty can be decreased by increasing the number of fitting data points.

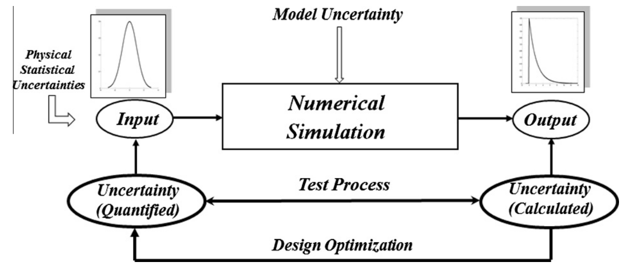

Fig. 1. Design under uncertainty.

Fig. 1 shows a simplified diagram for design under uncertainty. The design optimization process controls the input parameters (quantified uncertainties) presented by statistical diagram in order to satisfy the required output parameters (calculated uncertainties). The test process is a comparative process between the calculated output and the quantified input until convergence, as shown in Fig. 1.

Several strategies can be used for uncertainty measurements such as: Safety factor, Worst case scenario-convex models, Taguchi methods, Fuzzy set methods, Probabilistic methods... These strategies lead a high computing time to compute the probability of failure. An efficient optimization method based on reliability index can be easily implemented and perform the reliability analysis with a reasonable computing time.

2.2. Reliability index

To estimate the reliability index, several techniques have been developed during the last 40 years, namely FORM (First Order Reliability Methods), SORM (Second Order Reliability Method) and simulation techniques [6]. The image of the random variables in the standard normalized space is denoted $\boldsymbol{u}$, calculated by: $\boldsymbol{u} = T(\boldsymbol{x}, \boldsymbol{y})$ where $T(\boldsymbol{x}, \boldsymbol{y})$ is the probabilistic transformation function (Fig. 2). For a given failure scenario, the reliability index β is evaluated by solving a constrained minimization problem:

$$\beta = \min\ d(\mathbf{u}) \text{ subject to} : H(\mathbf{u}) = 0 \tag{1}$$

$$\text{with } d = \sqrt{\sum u_i^2} \tag{2}$$

where $\boldsymbol{u}$ is the vector modulus in the normalized space (or so-called distribution parameters), measured from the origin see Fig. 2. In FORM approximation, the probability of failure is simply evaluated by

$$P_f \approx \Phi(-\beta) \tag{3}$$

where $\Phi(\cdot)$ is the standard Gaussian cumulated function given as follows:

$$\Phi(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z} e^{-\frac{z^2}{2}} dz, \tag{4}$$

For practical engineering, Eq. (3) gives sufficiently accurate estimation of the failure probability.

The solution to problem (1) defines the Most Probable failure Point (MPP) see Fig. 2b. The resulting minimum distance between the limit state function $H(\boldsymbol{u}) = 0$ (Failure Surface) and the origin, is called the reliability index β [6].

3. Actual formulations for material properties

The mechanical properties of bone depend on composition and structure. However, composition is not constant in living tissues. It changes permanently in terms of the mechanical environment, ageing, disease, nutrition and other factors. Kopperdahl and