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# Analysis of the torsional strength of hardened splined shafts

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ABSTRACT

The current study presents a finite element modeling framework to determine the torsion strength of hardened splined shafts by taking into account the detailed geometry of the involute spline and the material gradation due to the hardness profile. The aim is to select a spline geometry and hardness depth that optimizes the static torsion strength. Six different spline geometries and seven different hardness profiles including non-hardened and through-hardened shafts have been considered. The results reveal that the torque causing yielding of induction hardened splined shafts is strongly dependent on the hardness depth and the geometry of the spline teeth. The results from the model agree well with experimental results found in the literature and reveal that an optimum hardness depth maximizing the torsional strength can be achieved if shafts are hardened to half their radius.

## 1. Introduction

There is an increased demand on power handling capacity of automotive transmission parts requiring high static and fatigue strength. Especially in splined shafts transferring high torque commonly used in heavy trucks, the choice of spline geometry and heat treatment process is crucial in obtaining an adequate strength of the component. For such connections yielding or ultimate fracture are potential modes of failure, particularly when overloads are involved. Since hardness of the material is strongly related to the strength of the material a common method of heat treatment of splined shafts is induction hardening, which increases the torque capacity of the shaft. Along with the material aspects associated with the heat treatment process, the strength of a spline connection is also dependent upon the geometrical design of the spline. Involute splines are commonly used in automotive industry with profiles similar to those of involute gear teeth. In the past, several studies [1–4] have focused on the failure analysis and failure prevention of splined or non-splined shafts; however none have attempted to quantify the simultaneous effect of the spline geometry and the hardness profile and depth on the torsion strength.

Several studies have focused on determining the fatigue behavior of splined shafts. In [5,6] the uniformity of load distribution on the spline teeth is studied by means of a finite element model. The model used accounts for the geometry of the splined shaft with the sleeve, from which it is found that the fatigue life of the shaft can be prolonged if a uniform load distribution can be assured. Determination of the residual stresses during induction hardening process of shafts is carried out in [6] by simulation and X-ray measurements. Results indicate that hardening depth has an influence on the residual stress level.

The objective in [7] is to improve the torsional fatigue life of a steel power transmission shafts. By varying the carburization process parameters such as soaking temperature and time different carburization depths are achieved. It is found that the carburization depth significantly influences the fatigue endurance limit under bending torsional loadings.

In [8] a numerical model was developed to simulate the induction hardening process of a steering pinion shaft by solving the coupled electromagnetic-thermal problem. They study the effects of induction parameters i.e. input AC current density, coil velocity and coil stay time with the model. The computed hardness profile curves show good agreement with the experimental data.

Similarly in [9–11] the authors studied experimentally the effects of induction parameters such as frequency, coil shape and applied current on final microstructure, the hardness of the work-pieces. Their investigation shows that the process parameters influence the residual stress-state of hardened parts to a great extent.

In [12] fatigue tests of induction hardened shafts are conducted in which failure occurs mainly due to subsurface crack initiation. Simulation results showed that the tensile residual stresses in the core increase with increased hardness depth, which is detrimental for fatigue life and a reason for subsurface crack initiation. The effect of torque overload on the fatigue life of barreled splined areoengine mainshaft couplings is investigated experimentally and numerically in [13,14]. A three dimensional finite element model is developed, however it does not take into account the material

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gradation resulting from the heat treatment process of the splined shaft. As reported in [15,16], engineering failure analysis and prevention of splined shaft failures is of great interest for the automotive industry when designing these components. However, none of the studies has so far addressed the combined effect of the geometry of the splined shaft and the material gradation in strength through the cross section of the shaft as a result of the induction hardening process.

Recently Kang et al. [17] performed finite element simulation of the induction hardening process of axle shafts with the objective to study the effect of surface hardening depth and distribution of residual stresses on the torsional strength of the shaft. The model they utilized is highly sophisticated taking into account the thermo-mechanical and metallurgical phase transformation behavior of the material during the induction hardening process. The shaft geometry they considered is circular in cross section, disregarding the effect of the spline design. Their results indicate that there is a strong effect of the induction hardening depth on the torsional strength. The importance of the hardening depth on the static and fatigue strength of induction hardened shafts has also been pointed out by Fett [18], where the relationship between induction hardening depth and torsional strength and fatigue life is determined though an extensive experimental program on induction hardened shafts of various material grades. Tortorella et al. [19] also performed static torsion experiments on induction hardened shafts and arrive upon the same conclusion as in [18] that the induction hardening depth has a paramount effect on the static strength.

Hence, the objective of this study is to develop a modeling framework which addresses some of the shortcomings of the models used in the literature to model the strength of hardened splined shafts. The analysis presented in this study aims to quantify the torsional strength of splined shafts by taking into account the detailed effect of the geometry of the spline and the gradation in material property, such as the hardness profile and hardness depth resulting from the hardening process.

## 2. Geometry of the involute spline

There are various international standards in which a thorough definition of the spline geometry can be found [20–22]. Here the geometry of the involute spline and the geometric quantities that define the spline will be briefly described. The terminology of an involute splined shaft is shown in Fig. 1. The pitch circle with radius $r_p$ is the theoretical circle upon which several geometric parameters are defined such as the pressure angle $\alpha$. The number of spline teeth is denoted $Z$ and together with $r_p$ defines the module $m$ of the spline as

$$m = 2r_p/Z \qquad (1)$$

which is an indication on the size of the spline tooth. The top and bottom land of the tooth are defined by the addendum radius $r_a$ and the dedendum radius $r_d$, respectively.

**Table 1**
Chemical composition of SAE 4140 (42CrMoS4).

| C (%) | Si (%) | Mn (%) | Cr (%) | Mo (%) |
|---|---|---|---|---|
| 0.40 | 0.25 | 0.85 | 1.00 | 0.25 |

In the present study six different spline geometries are considered comprising of two different pressure angles $\alpha = 30°$ and $45°$ and three different tooth numbers $Z = 20$, 30 and 40. Based on the polar moment of inertia of a circular shaft the static torsional yield strength scales with the cube of the spline dedendum radius ($r_d^3$). Therefore the addendum radius $r_a$ of the spline has been kept fixed for all cases while the dedendum radius $r_d$ is varied by changing the module $m$. Consequently the pitch radius $r_p$ is given by Eq. (1) once the module $m$ is set. The values of the module considered here are $m = 1.5$, 1.0 and 0.75 mm. These values for the module along with number of teeth given above gives a constant value of the pitch radius $r_p = 15$ mm.

## 3. Mechanical properties of SAE 4140 steel

The material considered in the shaft is a medium-carbon steel denoted SAE 4140 (42CrMoS4), which is commonly used in induction hardened transmission components and drive shafts in the automotive industry [23,24]. The chemical composition of SAE 4140 is given in Table 1.

By varying the process parameters in the induction hardening process various hardness depths and profiles can be achieved. Mechanical properties such as yield and ultimate strength are closely related to the hardness where the yield strength is approximately one third of the hardness in HV (Vicker's hardness). In order to establish a relation between stress–strain behavior and the hardness the materials simulation software JMATpro® [25] is utilized. The software is based on thermodynamics modeling combined with theoretical materials models and properties databases allowing for a quantitative calculation of wide range of thermophysical and mechanical quantities [26]. Given the chemical composition of a specific steel alloy, the stress–strain behavior can be determined with JMATpro®. In Fig. 2(a) the stress–strain curves show that the work-hardening behavior as well as the yield and ultimate strength strongly depend on the hardness level where each stress–strain curve correspond to a hardness value. Also the yield and ultimate strength for each hardness value can be

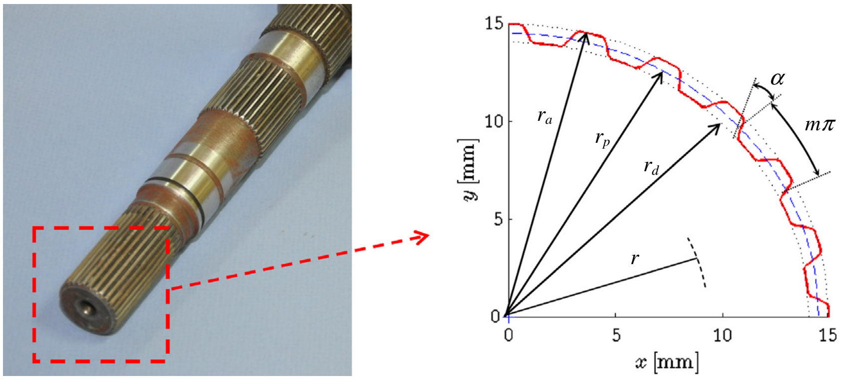

Fig. 1. Parameters defining the spline geometry.