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Short Communication

Determination of flow stress and the critical strain for the onset of dynamic recrystallization using a hyperbolic tangent function

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ABSTRACT

A new model has been developed to estimate the flow stress under hot deformation conditions up to the peak of the stress–strain curves. This model is derived from the general form of hyperbolic function by introducing an additional parameter to bring the results to a more acceptable level. Stress–strain curves and the critical strain of a '304 austenitic stainless steel' are determined with an average percentage error of 1.24. The model is also used to obtain an equation which has the ability of predicting the critical strain for the onset of dynamic recrystallization.

1. Introduction

While based on different physical theories, all the available models of flow stress curves are functions of strain, strain rate and temperature. Work hardening and dynamic recovery control the flow stress up to a maximum stress. However, dynamic recrystallization (DRX) occurs before this maximum value. There has been many efforts to determine the exact point of the onset of DRX experimentally, but there has been few attempts on determining this value mathematically [1–3].

Here, the flow stress is estimated by a new hyperbolic tangent function. From the mathematical point of view, the resulting equation could be used to predict the critical strain for the onset of dynamic recrystallization. This model is used to predict the stress–strain curves of '304 austenitic stainless steel' under different hot forming conditions. Computational results are highly consistent with the experiments.

2. Modeling of flow stress up to the peak

Taking a general form of hyperbolic tangent functions with an additional parameter (K_2), the following equation can be used for the prediction of flow stress curves up to the peak.

$$\sigma = \sigma_0 + (\sigma_P - \sigma_0)\left[\tan h\left(2\varsigma\frac{\varepsilon}{\varepsilon_P}\right)\right]^{K_2} \tag{1}$$

where σ is stress (σ_0 and σ_P are initial and peak stress, respectively), ε strain (ε_P is peak strain) and ς and K_2 are two material constants. This model results in Eqs. (2) and (3) which should be considered for determination of ς and K_2, respectively.

$$\tanh^{-1}\left(\frac{\sigma-\sigma_0}{\sigma_P-\sigma_0}\right)^{\frac{1}{K_2}} = \varsigma\left(2\frac{\varepsilon}{\varepsilon_P}\right) \tag{2}$$

$$\ln\left(\frac{\sigma-\sigma_0}{\sigma_P-\sigma_0}\right) = K_2 \ln\left[\tanh\left(2\varsigma\frac{\varepsilon}{\varepsilon_P}\right)\right] \tag{3}$$

These formulas reveal that the value of ς is the slope of the linear plot of $\tanh^{-1}\left(\frac{\sigma-\sigma_0}{\sigma_P-\sigma_0}\right)^{\frac{1}{K_2}}$ vs. $\left(2\frac{\varepsilon}{\varepsilon_P}\right)$ and K_2 the slope of the linear plot of $\ln\left(\frac{\sigma-\sigma_0}{\sigma_P-\sigma_0}\right)$ vs. $\ln\left[\tanh\left(2\varsigma\frac{\varepsilon}{\varepsilon_P}\right)\right]$. As can be seen, these variables are interdependent. In order to find these constants, different methods could be implemented. However, the simplest one is the iterative method which evaluates one constant and uses the calculated value to find the other one. The repetition of this procedure leads to two unique values.

Because Eq. (1) has two interdependent parameters, it has a powerful capability to predict the values of flow stress. It should be mentioned that it is the first constitutive equation with this kind of parameters.

3. Initiation of DRX

During hot deformation processes when the strain exceeds a critical value dynamic recrystallization begins with a driving force of removal of dislocations. Before the peak, the work hardening and dislocation density increase and result in a critical microstructural condition, i.e., new grains nucleate and new high-angle boundaries grow. Gradually, the dislocation density increases in other areas as well. Therefore, the flow stress increases, with a declining rate, up to a maximum value, and the rate of softening mechanism prevails over the work hardening afterwards. This phenomenon usually occurs for metals with low to medium stacking fault energy [4].

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4. Determination of the critical strain for the onset of dynamic recrystallization

Extensive reviews on DRX could be found in references [3,5,6]. The critical strain for the onset of DRX can be determined metallographically, which requires time-consuming and hard experiments. Therefore, researchers have tried to explain DRX mathematically. Perdrix and his colleagues examined the work hardening ($\theta = \frac{d\sigma}{d\varepsilon}$) curves and found that the $\theta - \sigma$ curve may be divided into successive linear portions with a negative slope up to the maximum point of the $\theta - \sigma$ curve ($\sigma = \sigma_P, \theta = 0$) [7]. Ryan and McQueen observed an inflection in the $\theta - \sigma$ curve where DRX is initiated [8]. Poliak and Jonas suggested that the minimum point of $\frac{d\theta}{d\sigma} - \sigma$ curve is where DRX is initiated [9].

Mathematically, a minimum point of $\frac{d\theta}{d\sigma} - \sigma$ is equivalent to a null value of the second derivative of θ with respect to σ and indicates the inflection of $\theta - \sigma$ curve [10].

$$\left.\frac{d^2\theta}{d\sigma^2}\right|_{critical\,strain} = 0 \quad (4)$$

Therefore, in order to solve Eq. (4) the first step is to find the derivative of Eq. (1):

$$\theta = 2\varsigma K_2 \frac{\sigma_P - \sigma_0}{\varepsilon_P} \tanh\left(2\varsigma\frac{\varepsilon}{\varepsilon_P}\right)^{K_2-1}\left(1 - \tanh\left(2\varsigma\frac{\varepsilon}{\varepsilon_P}\right)^2\right) \quad (5)$$

By solving Eq. (4) the critical strain is found as a function of the peak strain. The solution can be expressed as follows:

$$R_C^{DRX} = \frac{\varepsilon_C}{\varepsilon_P} = \frac{1}{\varsigma}\tanh^{-1}\left(\sqrt{\frac{1-K_2}{1+K_2}}\right) \quad (6)$$

5. Mathematical analysis of flow stress curves

The experimental results of hot torsion tests of '304 austenitic stainless steel' from the literature [11] were used for the mathematical analysis in the present investigation. Using MATLAB (The MathWorks, Inc., Natick, MA), a program was developed to solve Eqs. (2) and (3) in order to determine ς and K_2, respectively (see "Appendix A"). Empirically, five iterations were enough to reach converging results. Although it was expected that these constants were functions of temperature and strain rate, they were found to be independent from these hot deformation parameters (at least in this study). Therefore, mean values of these constants were used for further calculations which were determined to be 1.278 and 0.6772 for ς and K_2, respectively.

6. Discussion

Using Eq. (1), flow stress is calculated for all different hot deformation conditions and compared with experimental results (for about 200 data point). This comparison is illustrated in Fig. 1. The maximum, minimum and average percentage errors are evaluated as 8.66, 0.00 and 1.24, respectively. Besides, in order to find the coefficient of determination (R^2), a straight line is fitted to this diagram using the simple linear regression model. The formula and the R^2 value are reported in Fig. 1. The slope of the fitted line is 1.001 with a coefficient of determination of 0.9985. The low error values, the slope of the fitted line (≈1) and the high value of the coefficient of determination specify a high accordance between experimental and calculated flow stress.

Flow stress curves at different temperatures, for a strain rate of 1 s^{-1}, are depicted in Fig. 2. Obviously, the flow stress curves modeled by Eq. (1) demonstrate a good correlation with experiments, right from the initial stress up to the peak.

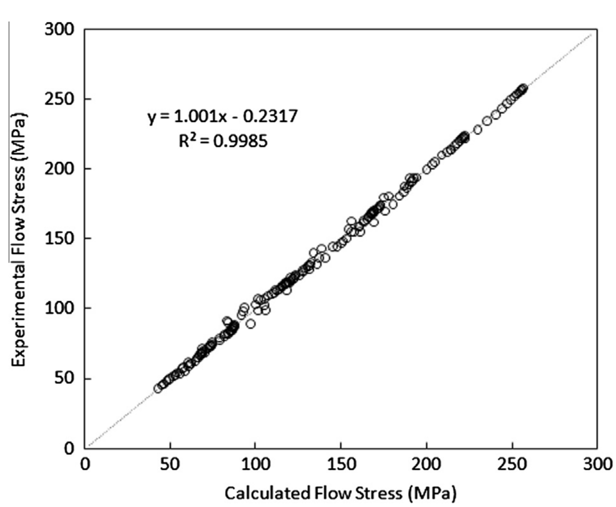

Fig. 1. Comparison between experimental and predicted flow stress values calculated by Eq. (1).

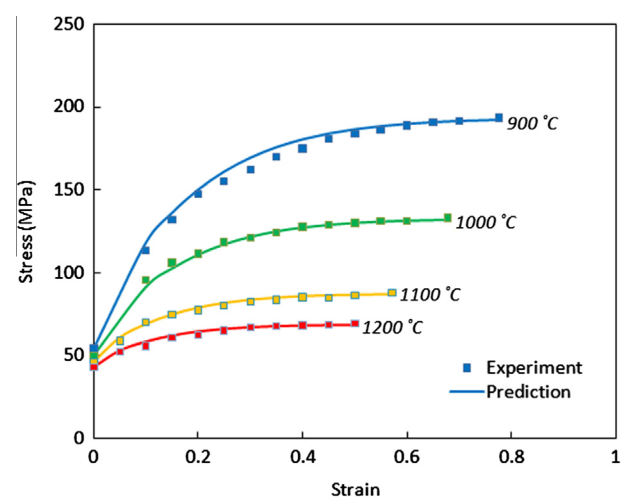

Fig. 2. Comparison between experimental and predicted flow stress curves for a strain rate of 1 s^{-1} at different temperatures.

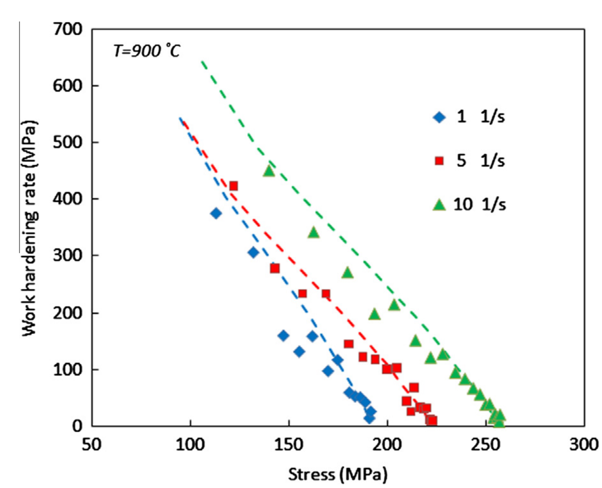

Fig. 3. Comparison between experimental (points) and predicted (hashed lines) work hardening plots at 900 °C with different strain rates.