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Theoretical analyses and numerical simulations on the mechanical strength of multilayers subjected to ring-on-ring tests

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ABSTRACT

Biaxial flexure tests have been applied widely to measure the strength of monolithic linear elastic materials. In spite of the increasing applications of multilayered composite materials, the theoretical description on their strengths using biaxial flexure tests seems very difficult and the relative fracture load-biaxial strength relationship for multilayered discs subjected to biaxial moment is still unavailable. In this paper, the advanced solutions for the elastic stress distribution in thin multilayered discs subjected to biaxial bending moment have been derived. We find the convertible relationship between monolayered and multilayered discs subjected to ring-on-ring tests. To evaluate the accuracy of the solutions, finite element analyses (FEA) are further performed on monolayered, bilayered with different thickness ratios and trilayered discs, respectively. In stress distribution results, we present stress gradient existing at the interface, which can harmonize the bilayered systems in our model. The present simple closed-form solutions can allow the biaxial strength of multilayered systems to be evaluated.

1. Introduction

In some fields, there are extensive applications of multilayered structures as optical, microelectronic, biological, and dental components. In order to keep the reliability and stability of multilayered systems, it is necessary to study the geometrical and material factors that affect their strength, and adopting the biaxial flexure tests to estimate the biaxial strength of multilayered systems has been performed. For the ceramic material, it has been used widely for circuit board and medicinal clinical restorations due to their advanced inertness and biocompatibility [1,2]. Generally, ceramic material has a high compressive strength, but it is very brittle and cannot resist high tensile stresses during loading [3,4]. Therefore, tensile strength becomes an important element that affects the success of clinical restorations [5]. There are two standard test methods used to measure flexure strength of ceramic materials. Uniaxial tensile tests, such as three-point bending of beam and four-point bending of beam, they have been applied to measure the strength of ceramics [6–12] in the past. Biaxial flexure tests, such as ball-on-ring, ball-on-three-ball, piston-on-ring, and ring-on-ring tests, are becoming popular as methods of determining the strength of ceramics [13–20]. In this case, a thin disc is supported by three balls (or a ring) near its periphery and equidistant from the center, also it is loaded through a piston, a smaller ring, or a ball in the central region. The area of maximum tension appears at the center of disc surface, where the stress is biaxial.

Especially, the piston-on-three-ball tests have been selected by international organization for standardization to establish ISO 6872 for the dentistry-ceramic materials [21]. In contrast, biaxial flexure tests have some advantages over uniaxial tensile tests. The measured biaxial strength does not depend on the edge condition and the edge failures are predicted difficultly for uniaxial tests [22], generally the real materials are subjected to multiple loading during applications, and then the data of biaxial strength become much useful for material design.

In a previous work, a solution of handling monolayer disc subjected to symmetrically distributed normal loadings was first developed by Nadai [23]. The relationship between transverse displacement and the load was represented by a biharmonic equation, its general solution could be acquired by using Muskhelishvili's complex variable method [24]. After solving the biharmonic equation by using the essential equilibrium and boundary conditions, Bassali [25] got a complex series solution, but due to the complexity of these formulations, application of these solutions is very difficult. To adapt in the design and analysis for thin discs easily, Bassali's solution was subsequently simplified by Vitman and Pukh [26] and Kirstein and Woolley [27], the results obtained from Roatk's formulas and Hsueh's solutions for bilayered discs were also included for comparison. The expression of the relationship between strength and fracture load for ring-on-ring tests are used

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only as the disc is monolithic [22,26–30] and ring-on-ring loadings on monolayered discs have been adopted by ASTM: C 1499-09, whereas the theoretical description for multilayered discs subjected to biaxial moment is unavailable. In addition, the effect of the disc with different thickness ratios on stress distribution through the thickness during tests and cracking origin are also lack. In order to solve these problems, in this paper, we will build the corresponding relationship between monolayered and multilayered discs subjected to ring-on-ring tests firstly, then the solutions of monolayer will be transformed to multilayered discs to systematically examine how the thickness ratio of discs affect the stress distribution through the thickness. Finally, we will analyze the cause of cracking and utilize the finite element analysis (FEA) to confirm the accuracy of the present solutions.

2. Materials and methods

2.1. Constitutive relations of multilayered discs

The stress analyses for biaxial flexure tests are very complex. It requires solving a biharmonic equation to characterize the relationship between the transverse displacement and the load for a disc, also it must satisfy the whole boundary conditions. At present, the existing widely analytical equations for monolayered disc have been simplified greatly, they can be adapted to the engineering and analysis for thin discs [26] easily. When the disc is a multilayered structure, the continuity conditions between discs need be analyzed, the analytical procedures are derived as follows.

An axial symmetry of thin elastic multilayered discs is shown schematically in Fig. 1. The disc is made up of n layers with individual thickness, t_i, where the subscript, i, indicates the layer number, the layer 1 locates at the bottom of the multilayered discs. The cylindrical coordinates r, θ, and z is used in the model. The bottom and top surfaces of multilayered discs are located at $z = 0$ and $z = h_n$, respectively. The interface between layers i and $i + 1$ is set as h_i. Thus h_n means the thickness of the disc, and the relation between h_i and t_i can be described by

$$h_i = \sum_{j=1}^{i} t_j \quad (i = 1 \text{ to } n) \tag{1}$$

The discs are subjected to ring-on-ring tests with $z = 0$ and $z = h_n$ as the supporting and the loading surfaces, respectively. In addition, the interfaces between discs are supposed to keep always bonded status during tests.

For a monolayered disc, the strains are proportional to the curvature of disc and the distance from the neutral surface, the neutral surface is in accord with the middle plane of the disc. But for multilayered discs, the neutral surface can deviate from the middle plane due to the discrepant elastic properties. In the model, the radial and tangential strains, ε_r and ε_θ are [31]:

$$\varepsilon_r = \frac{z - z_{nr}}{r_r} \quad (0 \leqslant z \leqslant h_n), \tag{2}$$

$$\varepsilon_\theta = \frac{z - z_{n\theta}}{r_\theta} \quad (0 \leqslant z \leqslant h_n), \tag{3}$$

where z_{nr} and $z_{n\theta}$ express the positions of neutral surfaces in the radial and the tangential directions, respectively, r_r and r_θ are the corresponding radius of curvature. Eqs. (2) and (3) describe the strain distribution in the model, while the stress normal can be neglected for a thin disc, the relations between stresses and strains are

$$\sigma_{ri} = E_i^p(\varepsilon_r + \nu_i \varepsilon_\theta) \quad (i = 1 \text{ to } n), \tag{4}$$

$$\sigma_{\theta i} = E_i^p(\varepsilon_\theta + \nu_i \varepsilon_r) \quad (i = 1 \text{ to } n), \tag{5}$$

where $E_i^p = E_i/(1 - \nu_i^2)$ is the plane-strain modulus, E is Young's modulus, ν is Poisson's ratio, and i means the layer number.

The relationships between stress and bending moment are

$$\sigma_{ri} = \frac{E_i^p(z - z_n^m)M_r}{D^m}, \tag{6}$$

$$\sigma_{\theta i} = \frac{E_i^p(z - z_n^m)M_\theta}{D^m}. \tag{7}$$

where M_r and M_θ are bending moments at two directions, and D^m is the flexural rigidity of multilayer, it is obtained by satisfying the force and the bending moment's equilibrium conditions,

$$D^m = \sum_{i=1}^{n} E_i^p t_i \left[h_{i-1}^2 + h_{i-1}t_i + \frac{t_i^2}{3} - z_n^m \left(h_{i-1} + \frac{t_i}{2} \right) \right], \tag{8}$$

The positions of the neutral surface can be acquired by satisfying the force equilibrium conditions, but the expression of z_{nr} and $z_{n\theta}$ become intricacy. If the difference between ν_i is neglected in Eqs. (4) and (5), the description of z_{nr} is simplified greatly. So bring $\nu_i = \nu$ into Eqs. (4) and (5), the expressions of z_{nr} and $z_{n\theta}$ are

$$z_n^m = z_{nr} = z_{n\theta} = \frac{\sum_{i=1}^{n} E_i^p t_i \left(h_{i-1} + \frac{t_i}{2} \right)}{\sum_{i=1}^{n} E_i^p t_i}. \tag{9}$$

The two neutral surfaces are equal and it is redefined as z_n^m. When a single disc has uniform material properties, $E_i = E$ and $\nu_i = \nu$, thus the relationships between stress and bending moment in monolayer disc are

$$\sigma_r = \frac{12(z - z_n)M_r}{h_n^3}, \tag{10}$$

$$\sigma_\theta = \frac{12(z - z_n)M_\theta}{h_n^3}, \tag{11}$$

Thus, Eqs. (8) and (9) become

$$D = \frac{Eh_n^3}{12(1 - \nu^2)}, \tag{12}$$

$$z_n = \frac{h_n}{2}. \tag{13}$$

where D and z_n show the flexural rigidity and the neutral surface of a monolayer disc. Compare Eqs. (6) and (7) with Eqs. (10) and (11), it is found that the stress distribution in multilayered discs must have some relations with that in monolayer disc. The descriptions of the relationships are

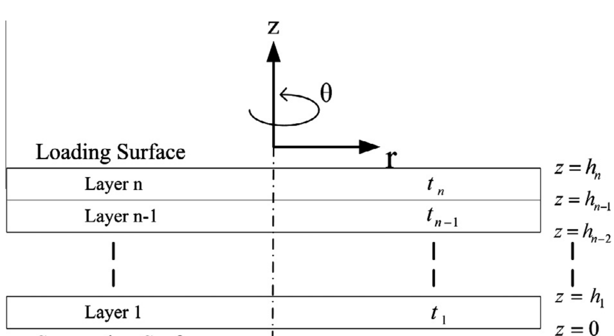

Fig. 1. Schematic of an axial symmetry of a thin elastic multilayered disc showing the coordinate system used.