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Short communication

A note on the *G*-function for needle leaf canopies

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Abstract

Formulas to calculate the radiation extinction coefficient or '*G*-function' for plant canopies originally were derived under the assumption of flat leaves. The *G*-function in that case is obtained as the ratio of the expected value (with respect to the leaf normal distribution function) of the projected leaf area to the one-sided leaf area. Although the corresponding formulas have later been presented also for conifer needles of different shapes they have rarely been used to determine conifer specific *G* values for parameterization of radiation models for coniferous stands. Instead, these models have typically been based on the erroneous assumption that the *G*-functions for flat leaves and needles are equivalent if only the one-sided leaf area (used as denominator in *G*) is replaced by the vertically projected, planimetric needle area. The importance of consistent definition of leaf area index and *G* for conifers is discussed in this study which presents an overview of previous studies on the *G*-function of needles and an easily applicable approach to calculate it for needles of different shapes.

Keywords: Extinction coefficient for radiation within canopy; Coniferous forest; Leaf area index; Leaf orientation; PAR

1. Introduction

Models describing the transfer of PAR in plant canopies typically are formulated in terms of the so-called gap probability (p_0) which describes the probability that a beam of radiation incident from a specified direction will pass through the canopy to reach a given point inside or below the canopy. In a horizontally uniform canopy with a completely random (Poisson) distribution of leaves, the gap probability at a given beam direction (solar elevation α) is given by

$$p_0 = \exp[-G(\alpha)L/\sin\alpha] \quad (1)$$

where L denotes the downward cumulative leaf area index and G is the extinction coefficient, also referred to as e.g., the 'mean projection of unit foliage area', the '*G*-function' or the mean projection coefficient (Ross and Nilson, 1965, Ross, 1981, Chen and Black, 1992). It is commonly acknowledged that the first to apply the Beer's law formula (Eq. (1)) in plant canopies were Monsi and Saeki (1953). Various modifications of the equation have been proposed in the numerous canopy radiation models that have been developed since then; however, the parameters L and G are involved in all of them. In the original definition of leaf area index (Watson, 1947), leaves were assumed to be flat (with zero thickness) and, logically, the area of a leaf was defined on a one-sided area basis. Parameter G, correspondingly, was defined as the mean ratio of projected to one-sided leaf area, where 'projected leaf area' refers to the sum of the shadow areas cast by leaves on a plane perpendicular to the beam direction. Obviously, the above definitions cannot be applied as such to coniferous canopies but need to be modified in the case of non-flat leaves. Traditionally, the projected area of needles

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as measured by planimeters (i.e. vertically projected area of horizontally lying needles) has been used as the counterpart to the one-sided area of flat leaves. To avoid confusion, this area could preferably be termed 'planimetric needle area' and the term 'projected needle area' reserved to denote the area appearing in the nominator of the G-function. As far as Eq. (1) is concerned, different definitions of L can be allowed as long as G is defined and calculated consistently. This means that the leaf/needle area in the denominator of G should be defined using the same basis as in the definition of leaf area index (L). However, the misconception that G values derived for flat leaves (see, e.g., Warren Wilson, 1967) apply to needles if only the one-sided leaf area (denominator of G) is replaced by the planimetric needle area still continues to cause confusion. Lang (1991, 1993) and Chen and Black (1992) among others have presented convincing arguments for the use of half of total or hemisurface needle area as basis for defining L of coniferous canopies – the discussion will not be repeated here but results presented will hopefully provide additional arguments for adopting that definition. Hemisurface needle area as the logical counterpart to one-sided flat leaf area was proposed already by Oker-Blom and Kellomäki (1981, 1982) who derived the extinction coefficient (G-function) of Scots pine needles, described as bisected cylinders. Later, a more extensive analysis including also other needle shapes was presented by Chen and Black (1992). However, although all the necessary information (theory and formulas) to calculate the projected area of needles of various shapes can be found in the literature (e.g., in the papers cited above), canopy specific G-values have rarely been determined for parameterization of radiation models for coniferous stands. Here, an attempt is made to overview and generalize previous results on the G-function of needles, and present an easily applicable approach to its determination.

2. The G-function of needle leaf canopies

Oker-Blom and Kellomäki (1981, 1982) derived the extinction coefficient (G-function) for needles whose shape was described as bisected cylinders. They showed also that the spherically averaged projection area of a needle is exactly one-fourth of its total surface area. As discovered by Lang (1991), the result actually applies to any object of convex shape according to a set of theorems proven already by Cauchy (1832). Similarly, as will be demonstrated, under certain not very restricting conditions the same G-function can be applied to needles of various different shapes.

The only assumptions concerning needle shape in the following analysis are that they are prismatic objects, i.e. with an invariant cross section perpendicular to the long axis (Chen and Black, 1992), and of convex shape. Let l_n denote the needle length, and let A_c and c denote the area and circumference of the needle cross section. The total needle surface area is then $l_n c + 2A_c$, and the hemisurface needle area is $A_n = l_n c/2 + A_c$. The projection area of a needle varies with needle orientation which, using the approach of Oker-Blom and Kellomäki (1982), is defined in terms of the inclination angle (τ) of the needle's long axis and the azimuth divergence angle (A) between the needle and Sun directions. In addition (except for rotationally symmetrical needles), the rotation angle (ν) around the needle's longitudinal axis is needed. The rotation angle can be mathematically described, e.g., with help of a vector normal to some fixed reference plane through the needle main axis. However, its formal definition is omitted here as the assumption made of a uniform rotation angle is believed to be intuitively clear.

G is defined as the expectation value of the projection area of a needle (EPA_n) on a plane perpendicular to the beam direction divided by the hemisurface needle area (A_n). Let β denote the (smaller) angle between the needle axis and the solar beam, which can be solved from

$$\cos\beta = \sin\alpha\sin\tau + \cos\alpha\cos\tau\cos A \qquad (2)$$

The expectation value of the projected needle (axis) length is $l_n E(\sin\beta)$. Assuming the rotation angle (ν) to be uniformly distributed, the projected needle width has the expectation value c/π (as for cylindrical needles). (The result follows from Cauchy's theorems; see also Oker-Blom and Kellomäki (1981) and Grace (1987).) We have now

$$EPA_n = l_n c E(\sin\beta)/\pi + A_c E(\cos\beta) \qquad (3)$$

where the second term on the right hand side is the contribution from the needle "tips", i.e. the cross sections at both ends.

The G-value, correspondingly, is obtained as

$$G = [l_n c E(\sin\beta)/\pi + A_c E(\cos\beta)]/(l_n c/2 + A_c) \qquad (4)$$

If A_c is very small in comparison to $l_n c$, as usually is the case for coniferous needles, the contribution from the needle tips can be neglected, and we arrive at the simple equation

$$G = 2E(\sin\beta)/\pi \qquad (5)$$