Clinica Chimica Acta 446 (2015) 213–217

Contents lists available at ScienceDirect

Clinica Chimica Acta

journal homepage: www.elsevier.com/locate/clinchim

Comparison of traditional, trimmed traditional and robust Youden charts

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ARTICLE INFO

Article history:
Received 10 December 2014
Received in revised form 8 April 2015
Accepted 10 April 2015
Available online 25 April 2015

Keywords:
Non-normal data
Statistical analysis
Statistical methods
Optimized analysis
Robust analysis
Youden chart

ABSTRACT

Background: Traditional Youden chart can be distorted by non-normally distributed data. Here, an optimized Youden chart was developed and compared with the traditional and trimmed traditional Youden charts.
Methods: The urea concentrations were determined by 28 laboratories to provide data for the construction of Youden charts. Normality of these data was tested. Outliers were excluded prior to construction of the trimmed traditional Youden chart. Non-robust and robust estimators were computed to construct the traditional and robust Youden charts, respectively. Robust between-laboratory *z*-score (ZB_i) and within-laboratory *z*-score (ZW_i) were obtained to assess whether or not these charts can reasonably present the urea results. Expected outcomes were the points related to acceptable ($|ZB_i|$ and $|ZW_i| \leq 2$), questionable ($2 < |ZB_i| < 3$ and/or $2 < |ZW_i| < 3$), and unacceptable ($|ZB_i|$ and/or $|ZW_i| \geq 3$) results fall inside, on/near, and outside the ellipse, respectively.
Results: Only the data from lot 201111 are non-normally distributed. Five- and 2-pair outliers are excluded from the data of lots 201111 and 201112, respectively. The concordance rates of the traditional, trimmed traditional and robust Youden charts are 87.1%, 92.9%, and 94.3%, respectively.
Conclusions: Among the three charts, the robust Youden chart presents the urea results best.

1. Introduction

In 1954, Dr Youden designed a new statistical method called the Youden chart, which is a 2-sample diagram for graphical illustration of inter-laboratory analytical results [1]. Youden emphasizes that the 2 samples should be similar and reasonably close in the magnitude [1]. In 1974, the Youden chart was modified by Mandel et al. by using an ellipse instead of a circle [2]. Mandel and Lashof have given detailed explanations of how to determine the orientation, major radius, minor radius, and center of the ellipse in different models [2]. The Youden chart has been successfully used in industry, agriculture, and medicine and has been recommended by the joint technical committee of the International Organization for Standardization and the International Electrotechnical Commission [3].

Essentially, a Youden chart is a scatter plot with a confidence region. It basically consists of an *x*–*y* axis, some data points, and an ellipse. Each data point corresponds to 2 analytical results of one laboratory defined by the first analytical result on the *x* axis and the second analytical result on the *y* axis. The ellipse is an approximate 95% confidence region. The locations of the points on the ellipse can illustrate which data points are acceptable, questionable, or unacceptable. Therefore, it is very important to construct a best fitting ellipse to correctly present these data. An ellipse is determined by three factors, "radii", "center", and "angle". The radii consist of the major and minor radii of the ellipse; these are the lengths of the 2 axes of the ellipse. The center is the center point of the ellipse and the angle is the orientation of the ellipse; this information determines the exact location of the ellipse. In fact, the best fitting ellipse is constructed by obtaining the most optimum radii, center, and angle. The radii, center, and angle of the traditional ellipse are derived from the non-robust estimators, thus non-normally distributed data can seriously distort the ellipse. This inspired us to determine the radii, center, and angle of the ellipse using trimmed non-robust estimators or robust estimators. The trimmed non-robust estimators are obtained using the same way for calculation of the non-robust estimators, but excluding outliers in the data. The robust estimators are computed using distribution-free inter-quartile range and median so that these estimators are more tolerant to non-normally distributed data.

In external quality assessment (EQA) programs, data usually are not normally distributed and sometimes the samples are small. Therefore, outliers in the data should be excluded or robust statistical techniques should be applied to deal with these data [4]. Because the Youden chart constructed by Mandel and Lashof is based on normality, it should be modified when used in the EQA program, by using trimmed non-

Abbreviations: EQA, external quality assessment; IQR, inter-quartile range; NIQR, normalized inter-quartile range.

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http://dx.doi.org/10.1016/j.cca.2015.04.021

robust estimators or robust estimators instead of non-robust estimators to determine the ellipse. In this study, actual EQA data are used to construct the traditional, trimmed traditional and robust Youden charts, the three charts are compared as well.

2. Materials and methods

2.1. Sample information

Data were obtained by analyzing the urea concentrations of serum samples obtained from an EQA program within the National Center for Clinical Laboratories in China. Samples of frozen serum with 5 different urea concentrations were provided by CLINIQA Corp. The samples were identified using lot numbers 201111–201115. The concentration range was approximately 0.70 to 49.15 mmol/l. The urea concentration of each sample was analyzed twice by the participants using a urease assay. The analytical procedure was the same as for the analysis of clinical samples. In total, 28 laboratories participated in this program; their analytical data are listed in Supplemental Table 1.

2.2. Test of normality

The normality of the urea data was tested with the Shapiro–Wilk test.

2.3. Exclusion of outliers

Before construction of the trimmed traditional Youden chart, the outliers in the data were excluded using a method of distribution-free inter-quartile range [4].

2.4. Calculation

2.4.1. Calculation of the estimators for the traditional Youden chart

The major radius (HL_p) and minor radius (HL_q) of the traditional ellipses were calculated using the formulas:

$$HL_p = \sqrt{2\lambda_1 Fc/(n-2)} \tag{1}$$

$$HL_q = \sqrt{2\lambda_2 Fc/(n-2)} \tag{2}$$

$$\lambda_1 = \left\{(S+Q) + \sqrt{(S-Q)^2 + 4D^2}\right\}/2 \tag{3}$$

$$\lambda_2 = \left\{(S+Q) - \sqrt{(S-Q)^2 + 4D^2}\right\}/2. \tag{4}$$

F_c is the critical value of F and n is the number of participants. Here, $F_{0.95}$ (2, 26) is 3.37 and n is 28 for all lot numbers.

The angle between the major axis of the traditional ellipse and the x axis (γ) was calculated using the formulas:

$$\tan\gamma = \left\{(S-Q) + \sqrt{(S-Q)^2 + 4D^2}\right\}/2D \; (\gamma \geq 0) \tag{5}$$

$$S = \sum y_i^2 - \left(\sum y_i\right)^2/n \tag{6}$$

$$Q = \sum x_i^2 - \left(\sum x_i\right)^2/n \tag{7}$$

$$D = \sum x_i y_i - \left(\sum x_i\right)\left(\sum y_i\right)/n. \tag{8}$$

The center of the traditional ellipse is the means of the first and second urea analytical results.

2.4.2. Calculation of the estimators for the trimmed traditional Youden chart

The major radius (HL'_p), minor radius (HL'_q), and angle (γ') of the trimmed traditional ellipse were calculated using the same formulas as above, however, the outliers in the data were excluded. Here, $F_{0.95}$ (2, 21) is 3.47 and n is 23 for lot 201111; $F_{0.95}$ (2, 24) is 3.40 and n is 26 for lot 201112; and $F_{0.95}$ (2, 26) is 3.37 and n is 28 for lot 201113–201115.

The center of the trimmed traditional ellipse is the means of the first and second urea analytical results from which the outliers are deleted.

2.4.3. Calculation of the estimators for the robust Youden chart

The major radius ($r\sigma_S$) and minor radius ($r\sigma_D$) of the robust ellipse were calculated using the formulas:

$$r\sigma_S = 2.448 \; \mathrm{NIQR}(S_i) \tag{9}$$

$$r\sigma_D = 2.448 \, \mathrm{NIQR}(D_i) \tag{10}$$

$$\mathrm{NIQR}(S_i) = 0.7413 \, \mathrm{IQR}(S_i) \tag{11}$$

$$\mathrm{NIQR}(D_i) = 0.7413 \, \mathrm{IQR}(D_i) \tag{12}$$

$$\mathrm{IQR}(S_i) = \mathrm{Q3}(S_i) - \mathrm{Q1}(S_i) \tag{13}$$

$$\mathrm{IQR}(D_i) = \mathrm{Q3}(D_i) - \mathrm{Q1}(D_i) \tag{14}$$

$$S_i = (x_i + y_i)/\sqrt{2} \tag{15}$$

$$D_i = (x_i - y_i)/\sqrt{2}. \tag{16}$$

$\mathrm{NIQR}(S_i)$ and $\mathrm{NIQR}(D_i)$ are the normalized inter-quartile ranges of the standardized sum and difference, respectively; $\mathrm{IQR}(S_i)$ and $\mathrm{IQR}(D_i)$ are the inter-quartile ranges of the standardized sum and difference, respectively; $\mathrm{Q3}(S_i)$ and $\mathrm{Q3}(D_i)$ are the upper quartiles of the standardized sum and difference, respectively; and $\mathrm{Q1}(S_i)$ and $\mathrm{Q1}(D_i)$ are the lower quartiles of the standardized sum and difference, respectively. x_i and y_i are the first and second analytical results of laboratory i, respectively. The factor 2.448 means that a 95% confidence ellipse can be obtained when the lengths of the radii are 2.448 standard deviations [1]. The normalized factor 0.7413 comes from the standard normal distribution [5].

The angle between the major axis of the robust ellipse and the x axis (δ) was fixed to 0.785 rad (45°).

The center of the robust ellipse is the medians of the first and second urea analytical results.

2.4.4. Calculation of the robust z-score

The robust between-laboratory z-score (ZB_i) and within-laboratory z-score (ZW_i) were calculated based on the formulas:

$$\mathrm{ZB}_i = \mathrm{S}_i - \mathrm{med}(S_i)/\mathrm{NIQR}(S_i) \tag{17}$$

$$\mathrm{ZW}_i = \mathrm{D}_i - \mathrm{med}(D_i)/\mathrm{NIQR}(D_i) \tag{18}$$

$\mathrm{med}(S_i)$ and $\mathrm{med}(D_i)$ are the medians of the standardized sum and difference, respectively [5].

2.5. Comparison of the traditional, trimmed traditional and robust Youden charts

The comparison was based on the z-score and the location of its corresponding point on the ellipse. The expected outcomes are: $|\mathrm{ZB}_i|$ and $|\mathrm{ZW}_i| \leq 2$, the corresponding point falls inside the ellipse; $2 < |\mathrm{ZB}_i| < 3$ and/or $2 < |\mathrm{ZW}_i| < 3$, the corresponding point falls on or near the ellipse; and $|\mathrm{ZB}_i|$ and/or $|\mathrm{ZW}_i| \geq 3$, the corresponding point falls outside the ellipse.