Materials and Design 32 (2011) 3876–3882

Contents lists available at ScienceDirect

Materials and Design

journal homepage: www.elsevier.com/locate/matdes

A notch strain calculation of a notched specimen under axial-torsion loadings

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ARTICLE INFO

Article history:
Received 14 December 2010
Accepted 1 March 2011
Available online 5 March 2011

Keywords:
A. Ferrous metals and alloys
F. Plastic behavior
H. Failure analysis

ABSTRACT

In this paper, a notch analysis model is presented for the numerical prediction of multiaxial strains of a notched 1070 steel specimen under combined axial and torsion loadings. The proposed model is based on the notion of a structural yield surface and uses a small-strain cyclic plasticity model to describe stress–strain relations. A notch load–strain curve is calculated with Neuber's rule and incremental nonlinear finite element analysis. The presented model is applied to simulate the notch root deformations of a circumferentially notched specimen under cyclic tension–compression–torsion loading histories. The model predictions are evaluated with strain measurements at the notch root of the specimen in a comprehensive set of cyclic tests. The computed strain loops were in accord with experimental data and matched qualitatively with measured shear–axial strain histories irrespective of loading path of the test. In proportional balanced torsion-axial loading, the nonlinear shear strain–axial strain loops were calculated properly. The modeling errors were determined to be a function of the loading path shape, and compared to shear strains, axial strain predictions were more accurate.

1. Introduction

The constantly reducing lead-times and development expenses in automotive and related industries put more rigorous restrictions on the design development and verification practices of automotive components. Developing a fatigue-safe component design is probably the one of the most critical ingredients of manufacture of durable products. Therefore, engineers have devised both experimental and analytical methods to estimate the useful design life of structural parts and to assess associated fatigue damage under multiaxial fatigue loads at service conditions [1]. Since the design features such as fillets, welds, shoulders, generally known as the notch, are commonly the potential sides of component failures [2]; a proper investigation of local material deformations at such geometrical discontinuities is essential in both fatigue testing and analytical fatigue analyses in this context.

Due to the multiaxial stress state caused by the notch constraint, closed-formed analytical solutions do not exit to calculate inelastic stress–strain response at notches. The numerical modeling, in particular finite element (FE) analyses, provide a reliable solution that can be applied in the variety of component geometry [3,4], but the applicability of this approach depends on computer modeling of the testing process. The FE analysis costs should also be considered especially for cases where elasto-plastic notch deformations necessitate incremental FE solutions for relatively long loading histories [3]. Alternative to FE analysis, notch analysis methods have been introduced to calculate the stress and strain components at a single material point, usually the notch root [5]. These analytical models seek approximate stress–strain solutions under plane stress conditions and use the elastic stresses calculated with elasticity theory as the fundamental input. The notch stress–strain solutions by notch analysis methods are exact for elastic material deformations, but in the case of elasto-plastic notch deformations, additional equations besides plasticity relations are required and the calculated notch stresses and strains are approximate under general loading conditions. Compared to FE analyses, however, the notch analysis methods are practical in terms of computational efficiency and operational expenses, and consequently, engineers have devised different methods to estimate the fatigue life of structural parts under dynamic loading conditions [6–10].

In this study, a notch analysis model is presented for estimation of elasto-plastic notch stress and strains under proportional and nonproportional cyclic loading conditions. The proposed model uses elastically calculated notch stress history as the basic model input and effective stress/strain measures to extend the Neuber's rule to multiaxial stress states [11]. The small-strain cyclic plasticity model developed by Chaboche [12] is employed to describe the notch plasticity relations. The model is applied to simulate the notch root deformations of a circumferentially notched specimen loaded by cyclic tension–compression–torsion loading histories. The model predictions are compared with the measured notch strains determined with a comprehensive set of axial force-torsion testing programs.

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doi:10.1016/j.matdes.2011.03.005

2. Multiaxial notch analysis

The notch geometries studied in earlier literature were two-dimensional leading to uniaxial stress state at the notch root, and proportional monotonic loading conditions were considered [2,4,5,11]. Neuber [13] studied a semi-infinitely prismatic body under anti-plane shear loadings and developed an expression relating the theoretical stress concentration factor to the elastic–plastic stress and strain concentration factors. Molski and Glinka [14] proposed a uniaxial approximation formula for a notched body under plane stress conditions based on equivalence of strain energy density of two identical bodies made of ideally elastic and elastic–plastic materials. Hoffmann [15] and Hoffmann and Seeger [16] proposed a multiaxial extension of Neuber's rule by replacing the notch and nominal stress–strain quantities with respective equivalent forms, and reported successful notch root stress–strain calculations for round bars with mild, sharp and sharp-deep notches. Moftakhar [17] analyzed two materials, one ideally elastic and the other elastic–plastic, and showed that the total strain energy density at the notch tip of an elastic body is greater under a monotonically increasing multiaxial loading. Barkey [18] introduced the notion of structural yield surface in relating nominal stresses to the notch strains based on a structural constitutive relation and employed a multiple-surface plasticity model using the elastically calculated nominal stresses as the local loading inputs. Koettgen [19] introduced the notion of pseudo stress–strain for the sake of simplified nominal stress or strain definitions for arbitrary geometries, and replaced the Barkey's anisotropic structural yield surface with the matrix of elastically-calculated scaling factors. The elastically computed stresses are input to the structural plasticity algorithm to compute notch strains via a notch load–strain or a pseudo stress–notch strain curve. Computed notch strains were employed in the cyclic plasticity model to obtain notch stresses within the subsequent stage. Koettgen et al. [20] reported good correlations of the notch strains for various shaft geometries when compared with elastic–plastic FE analyses, and similar conclusions are drawn by other researchers employing different plasticity models for the sake of computational efficacy [21–23]. In a recent research, Ye and his coworkers [24] derived an unified expression for the strain energy densities considering the stored and dissipated energies at the notch root. Their notch analysis model combines Neuber's rule and Monski-Glinka's energy criterion in an energy balance expression and employs three incremental strain ratio expressions following Moftakhar's approach.

Based on the review of previous studies in the literature, the cyclic plasticity modeling utilizing the structural yield surface concept is chosen as an efficient methodology in the numerical prediction of multiaxial notch deformations under cyclic loading conditions. The small-strain plasticity model proposed by Chaboche [12] is implemented to describe notch plasticity and stress–strain relations. The structural yield surface is defined in the pseudo stress space by means of the von Mises yield function. The evolving anisotropy of plastic deformation is defined by the translation of yield surface following a nonlinear kinematic hardening rule. The multiaxial loading is described by the history of pseudo stress tensor at a single material point and input to the structural plasticity model. The pseudo stresses are nothing but a fictitious tensor quantity computed with the theory of elasticity for a single material point [20]. Small deformations are assumed and the anisotropy of yield surface is modeled with a matrix of scaling constants. Considering a set of M different external loads acting on the component, the pseudo-stress tensor ${}^e\sigma_{ij}$ is the superposition of a set of M stress tensors equal to the elastic stress tensor calculated for each external loads acting on the component separately.

$$ {}^e\sigma_{ij} = \sum_{m=1}^{M} (C_{ij})_m L_m \tag{1}$$

where $(C_{ij})_m$ are scaling coefficients that are equal to the elastic stress tensor calculated for each single external load L_m with unit magnitude. To relate the local loads to the elastic–plastic response at a structural point, a pseudo stress–notch strain curve is employed. The pseudo stress–notch strain curve may be generated by FE analyses or by a uniaxial approximation formula such as Neuber's rule [13] or equivalent strain energy density method [14]. The application details of both methods may be found in respective works in literature [15–17,20,22–24].

2.1. Multiaxial stress–strain analysis

A rate-independent plasticity model using nonlinear kinematic hardening rule is employed to calculate the stress–strain history. A brief description of the model is given below, and the detailed mathematical formulation can be found in [12]. Small deformations and additive decomposition of total strain as elastic and plastic parts are assumed. Elastic deformations follow Hooke's law until the yield condition is satisfied. The yield function is expressed as:

$$ f = J_2(\underline{\underline{\sigma}} - \underline{\underline{X}}) - k \tag{2}$$

where J_2 is the second invariant of deviatoric relative stress, and k is the yield stress in simple shear. $\underline{\underline{X}}$ represents the total backstress composed of m parts. The shape and the orientation of yield surface in stress space are assumed not to change, and the size of yield surface may be changed to account transient effects. The evaluation equation for the increment of backstress parts is expressed as,

$$ d\underline{\underline{X}}^{(i)} = \frac{2}{3} C^{(i)} d\underline{\underline{\varepsilon}}^p - \gamma^{(i)} \underline{\underline{X}}^{(i)} dp \tag{3}$$

where $C^{(i)}$ and $\gamma^{(i)}$ are material parameters, and dp is the increment of accumulative plastic strain. The normality hypothesis and the consistency conditions leads to the expression of hardening modulus h as the sum of hardening modulus from each backstress parts.

$$ h = \sum_{i=1}^{m} h^{(i)} \tag{4}$$

and,

$$ h^{(i)} = C^{(i)} - \frac{3}{2} \gamma^{(i)} \underline{\underline{X}}^{(i)} : \frac{\underline{\underline{\sigma}}' - \underline{\underline{X}}'}{k} \tag{5}$$

Assuming normality rule the expression of increment of plastic strain tensor is derived by using the plastic potential function F.

$$ F = J_2(\underline{\underline{\sigma}} - \underline{\underline{X}}) + \frac{3}{4} \sum_{i=1}^{m} \frac{\gamma^{(i)}}{C^{(i)}} \underline{\underline{X}}^{(i)} : \underline{\underline{X}}^{(i)} \tag{6}$$

$$ d\underline{\underline{\varepsilon}}^p = \frac{\partial F}{\partial \underline{\underline{\sigma}}} : dp = \frac{1}{h} \left\langle \frac{\partial f}{\partial \underline{\underline{\sigma}}} : d\underline{\underline{\sigma}} \right\rangle \frac{\partial f}{\partial \underline{\underline{\sigma}}} \tag{7}$$

$$ d\underline{\underline{\varepsilon}}^p = \frac{2}{3} \frac{\underline{\underline{\sigma}}' - \underline{\underline{X}}'}{J_2(\underline{\underline{\sigma}} - \underline{\underline{X}})} dp \tag{8}$$

The material parameters $C^{(i)}$ and $\gamma^{(i)}$ are computed using the cyclic stress–strain curve of the material. Depending on the strain and stress amplitudes, Masing and non-Masing behaviors can be simulated under cyclic balanced loading conditions. Jiang and Sehitoglu [25] proposed a general method in the computation of material parameters in the Armstrong–Frederich type of backstress evaluation, and this approach is employed in this study. The com-