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Thermal stability analysis of circular functionally graded sandwich plates of variable thickness using pseudo-spectral method

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ABSTRACT

In the present study, the thermal stability of laminated functionally graded (FGM) circular plates of variable thickness subjected to uniform temperature rise based on the first-order shear deformation plate theory is presented. Furthermore, two models for FGM plates with variable thickness, corresponding with two manufacturing methods, are proposed. The laminated FGM plate with variable thickness is considered as a sandwich plate constituted of a homogeneous core of variable thickness and two constant thickness FGM face sheets whose material properties are assumed to be graded in the thickness direction according to a simple power law. In order to determine the distribution of the prebuckling thermal load along the radius, the membrane equation is solved using the shooting method. Subsequently, employing the pseudo-spectral method that makes use of Chebyshev polynomials, the stability equations are solved numerically to evaluate the critical temperature rise. The results demonstrate that the thermal stability is significantly influenced by the thickness variation profile, aspect ratio, the volume fraction index, and the core-to-face sheet thickness ratio.

1. Introduction

Stability analysis and studies on the buckling behavior of plates have always been considered as one of the important subjects in structural analysis (Timoshenko and Gere [1], Almroth and Brush [2], and Turvey and Marshall [3]). On the other hand, variable thickness plates have always been attractive for designers, and a lot of researches have been done on this subject. The most conspicuous usage of variable thickness plates is to lighten structures, especially when used in high-speed aircrafts. With an accurate design of the thickness distribution, one can make an increase in buckling capacity of the plate compared to its uniform thickness counterpart. Wang et al. [4] investigated the elastic buckling of tapered circular plates based on the shooting method and the Rayleigh–Ritz approach. Raju and Rao [5] evaluated the post-buckling behavior of linearly tapered moderately thick isotropic circular plates subjected to thermal load by employing a general finite element formulation. Özakça et al. [6] carried out the buckling analysis of tapered circular and annular plates using the finite element method. A family of variable thickness, Mindlin–Reissner axisymmetric finite elements is developed which include shear deformation and rotary inertia effects. Shufrin and Eisenberger [7] studied the buckling behavior of thick elastic rectangular plates with variable thickness, applying both the first-order and high order shear deformation plate theories, using the extended Kantorovich method.

Functionally graded materials (FGM) are a group of composite materials whose properties vary continuously from one side to another. The concept of FGM was first introduced by a group of material scientists in Japan in 1984 [8]. These materials are typically constructed from a mixture of ceramic and metal and they can survive environments with high-temperature gradients such as nuclear reactors and high-speed aircrafts. The low thermal conductivity of ceramic provides the high-temperature resistance. On the other hand, the ductile metal prevents fracture caused by thermal stresses. Aboudi et al. [9] used a higher-order theory for the response of a functionally graded composite plate subjected to a through-thickness thermal gradient to optimize the composite's microstructure. Praveen and Reddy [10] investigated the static and dynamic response of functionally graded ceramic–metal plates, using a plate finite element that accounts for the transverse shear strains, rotary inertia and moderately large rotations in the von-Karman sense, and discussed the effect of the imposed temperature field on the response of the FGM plate. Najafizadeh and Eslami [11] analyzed the thermal buckling of FGM circular plates under uniform temperature rise, thermal gradient across the thickness, and thermal gradient across the radius. Ma and Wang [12] investigated axisymmetric large deflection bending and thermal post-buckling behavior of an FGM circular plate, based on the

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classical non-linear von-Karman plate theory, using a shooting method. Shen and Li [13] studied compressive post-buckling under thermal environments and thermal post-buckling due to heat conduction for a simply supported, sandwich plate with FGM face sheets having temperature-dependent properties, using a two-step perturbation technique and based on a higher order shear deformation plate theory. The non-linear behavior of FGM plates exposed to a high-temperature environment using first-order shear deformation theory is investigated by Prakash et al. and non-linear governing equations derived based on von-Karman assumptions are solved using Newton–Raphson technique [14]. Na and Kim [15] carried out the three-dimensional thermomechanical buckling analysis for FGM composite structures that composed of ceramic and metal layers, using the finite element method and assuming the material properties to be temperature dependent. Najafizadeh and Heydari [16] considered thermal buckling of FGM circular plate under thermal loads based on the higher order shear deformation theory. Zhao et al. [17] reported the mechanical and thermal buckling analysis of square FGM plates including plates that contain square and circular holes at the center, based on the first-order shear deformation plate theory, using the mesh-free method.

Recently, the subject of FGM plates with variable thickness has been taken under advisement by researchers. Xiang and Yang [18] presented the free and forced vibration of a laminated functionally graded Timoshenko beam of variable thickness, which consists of a homogeneous substrate and two inhomogeneous functionally graded layers, subjected to one-dimensional steady heat conduction in the thickness direction, employing the Differential Quadrature (DQ) method. Exact vibration analysis of an FGM annular plate of variable thickness, using the first-order shear deformation plate theory is presented using the exact element method by Efraim and Eisenberger [19]. Xu and Zhou [20] investigated the stress and displacement distributions of continuously varying thickness functionally graded rectangular plates simply supported at four edges. Applying the finite element method, Naei et al. [21] studied the buckling analysis of radially-loaded FGM circular plates with variable thickness using energy method based on Love–Kirchhoff hypothesis and Sander's non-linear strain–displacement relation for thin plates.

FGM plates would constitute a significant part of structural applications in future. On the other hand, optimum design is a big concern especially in applications such as aerospace, where reducing the structural members' weight is essential. Considering these facts demonstrates the importance of researches dealing with the behavior of FGM plates with variable thickness. However, with regard to the aforementioned researches, it is obvious that the analysis of FGM plates with variable thickness, especially in the field of stability problems, has not been noticed very much. Therefore, in this study, the thermal stability of FGM circular plates with variable thickness under temperature rise is investigated for clamped and simply supported boundary conditions. The equilibrium and associated stability equations are derived based on the first-order shear deformation plate theory in the von-Karman sense. Furthermore, two models for FGM plates with variable thickness, corresponding two manufacturing methods, are proposed.

2. Problem formulations

2.1. FGM plates with variable thickness

In FGM plates, material properties typically vary in the thickness direction based on the change in the volume fraction of the components which itself depends on the value of thickness and the distance from the surface in the thickness direction. Consider an FGM circular plate where r and z define the radial and thickness directions, respectively. If the thickness is constant, the volume fraction will be the same for all sections in the radial direction and it will only be a function of z. But if the thickness is variable, the value of thickness will differ in the r direction and the volume fraction definition will be more complicated.

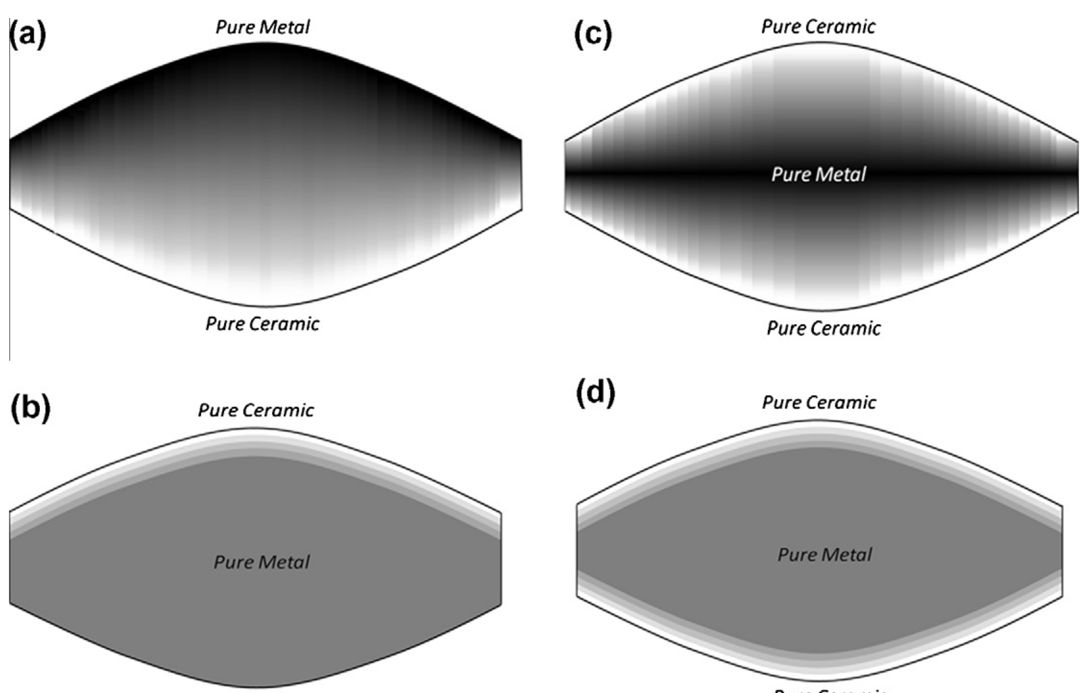

Fig. 1. Different types of FGM circular plates with variable thickness. (a) A one-sided FGM plate with the volume fraction depending on thickness. (b) A one-sided FGM plate with constant thickness FGM layer. (c) An FGM bimorph plate with the volume fraction depending on thickness. (d) An FGM bimorph plate with constant thickness FGM layers.