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Plasticity models for concrete material based on different criteria including Bresler–Pister

Tayfun Dede *, Yusuf Ayvaz

Department of Civil Engineering, Karadeniz Technical University, 61080 Trabzon, Turkey

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ABSTRACT

The purpose of this study is to investigate nonlinear behavior of reinforced concrete (RC) structures with the plasticity modeling. For this aim, a nonlinear finite element analysis program is coded in MATLAB. This program contains several yield criteria and stress–strain relationship for compression and tension behavior of concrete. In this paper, the well-known criteria, Drucker–Prager, von Mises, and Mohr Coulomb, and a new criterion-Bresler–Pister are taken into account. The elastic–perfectly plastic and Saenz stress–strain relationships in compression and tension stiffening in tension behavior of concrete are used with four different yield criteria mentioned above. The proposed models are in good agreement with the experimental and analytical results taken from the literature. It is concluded that the coded program, the proposed models, and Bresler–Pister criterion can be effectively used in nonlinear analysis of reinforced concrete beams.

1. Introduction

Considerable constitutive models have been proposed to define nonlinear behavior and stress–strain relationship of reinforced concrete (RC) material. These models can be classified into orthotropic models, nonlinear elastic models, plasticity models, endochronic models, fracture mechanics models and micromodels [1]. Using these models, several studies have been made in the field of nonlinear analysis of RC structure to predict the behavior of reinforced concrete structures more reliable. Arslan [2] investigated the sensitive of the Drucker–Prager modeling parameters and the use of it in plasticity theory for shear design of RC beams. Park and Klingner [3] presented a nonlinear analysis study of RC members by using plasticity multiple failure criteria. Wang and Hsu [4] applied the nonlinear finite element analysis to various types of RC structures using a new set of constitutive models. Bratina et al. [5] presented a study on materially and geometrically nonlinear analysis of RC planar frames by dealing with the fiber-based constitutive equations of concrete and steel. Zhao et al. [6] studied the load-deflection and failure characteristics of deep RC coupling beams. Pankaj and Lin [7] used two similar continuum plasticity material models to examine the influence of the material modeling on the seismic response of RC frame structures. Belmouden and Lestuzzi [8] investigated post peak modeling and nonlinear performance of RC structural walls. Bischoff [9,10], Stramandinoli and Rovere [11] and Dede and Ayvaz [12] studied on RC structures by considering tension stiffening effect.

Among the models given above, plasticity models need a yield function, a hardening rule, a flow rule and a stress–strain relationship to construct the plastic material matrix for the plastic behavior of concrete. A review of the literature indicates that there are not any studies based on the Bresler–Pister criterion for plastic behavior of concrete. This yield function can be found in the books concerning with the plasticity theory. But, its plasticity material matrix or any application of this function to the RC structures is not found.

In this paper, derivation of plastic material matrix based on Bresler–Pister yield function and two applications of this function to the RC beams are presented. For this aim, a nonlinear finite element analysis program is coded in MATLAB. This program contains several yield criteria and stress–strain relationship for compressive and tensile behavior of concrete. In the nonlinear analysis, the well-known criteria, Drucker–Prager, von Mises, and Mohr Coulomb and as a new criterion, Bresler–Pister, are taken into account. The elastic–perfectly plastic and Saenz stress–strain relationship in compressive and tension stiffening in tensile behavior of concrete are used with four different yield criteria mentioned above.

2. Yield criteria for concrete

The concrete is assumed to be elastic until it reaches the yield limit. Beyond yielding, plastic deformations take place. So, residual plastic deformations remain after removing the loading. A considerable amount of formulations have been proposed for concrete as

* Corresponding author. Tel.: +90 462 3772638; fax: +90 462 3772606.
E-mail address: tayfundede@gmail.com (T. Dede).

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Nomenclature

f'_c	uniaxial compressive cylinder strength	D	elastic material-stiffness tensor
f'_t	uniaxial tensile strength	D_c	material matrix of concrete
θ	angle of similarity	D^{ep}	elastic–plastic material-stiffness tensor
τ	effective von Mises stress	D^p	plastic material-stiffness tensor
ϕ	internal friction angle	D_s	material matrix of equivalent reinforcing bar elements
δ	kronecker delta	E	Young's modulus
α, k	material parameters	f	yield function
ε_{cr}	cracking strain of concrete	H^p	plastic hardening modulus
σ_{cr}	cracking stress of concrete	I_1	first invariant of stress tensor
σ_f, ε_f	control point coordinates on stress–strain curve	J_2	second invariant of stress deviator tensor
σ_{oct}	octahedral normal stress	J_3	third invariant of deviatoric stress tensor
τ_{oct}	octahedral shear stress	K	initial tangent modulus
ε_p	concrete strain corresponding to σ_p	s	deviatoric stress
σ_p	peak concrete compressive stress	ε	strain
ρ_x	reinforcement ratio in global direction of the X axis	ε^e	elastic strain
ρ_y	reinforcement ratio in global direction of the Y axis	ε^p	plastic strain
c	cohesion	σ	stress

a yield function such as Drucker–Prager, von Mises, Mohr Coulomb, Tresca, Rankine, William Warnke, Ottosen, Hsieh Ting Chen, and Bresler–Pister [13]. The well-known yield function for Drucker–Prager, von Mises, and Mohr Coulomb are given by the following equations, respectively [14].

$$f = \alpha I_1 + \sqrt{J_2} - k \tag{1}$$

$$f = \sqrt{J_2} - k \tag{2}$$

$$f = I_1 \sin\phi + \frac{1}{2}\left[3(1-\sin\phi)\sin\theta + \sqrt{3}(3+\sin\phi)\cos\theta\right]\sqrt{J_2} - 3c\cos\phi \tag{3}$$

$$\cos(3\theta) = \frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}. \tag{4}$$

where f is yield function, α and k are the material parameters, c is cohesion, ϕ is internal friction angle, I_1 is the first invariant of stress tensor, J_2 is the second invariant of deviator stress tensor, J_3 is the third invariant of deviator stress tensor, and θ is angle of similarity.

The Bresler–Pister criterion is the extension of Drucker–Prager criteria. This yield function in terms of octahedral stresses is given by

$$\frac{\tau_{\text{oct}}}{f'_c} = a - b\left(\frac{\sigma_{\text{oct}}}{f'_c}\right) + c\left(\frac{\sigma_{\text{oct}}}{f'_c}\right)^2. \tag{5}$$

where a, b, and c are the material parameters of this yield function. These parameters can be established by using available experimental test data given in Table 1 [14]. In this table, $\bar{f}'_t$ and $\bar{f}'_{bc}$ are the normalized strengths, f'_t is uniaxial tensile strength, f'_c is uniaxial compressive cylinder strength, f'_{bc} is equal biaxial compressive strength, σ_{oct} is octahedral normal stress and τ_{oct} is octahedral shear stress.

The octahedral normal and shear stresses are given by the following equations, respectively.

$$\tau_{\text{oct}} = \sqrt{\frac{2}{3}J_2} \qquad \sigma_{\text{oct}} = \frac{I_1}{3} \tag{6}$$

The normalized strengths are given by the following equations.

$$\bar{f}'_t = \frac{f'_t}{f'_c}, \quad \bar{f}'_{bc} = \frac{f'_{bc}}{f'_c}. \tag{7}$$

When these experimental test data are substituted into Eq. (5), the parameters a, b and c can be obtained by solving a system of three linear equations given below.

$$\begin{aligned} a &= \frac{\sqrt{2}}{3}\bar{f}'_t\bar{f}'_{bc}(8\bar{f}'_{bc} + \bar{f}'_t - 3)/\Delta \\ b &= \sqrt{2}(4\bar{f}'^2_{bc} - \bar{f}'_{bc} - \bar{f}'_{bc}\bar{f}'_t + \bar{f}'_t)(1 - \bar{f}'_t)/\Delta \\ c &= 3\sqrt{2}(3\bar{f}'_t\bar{f}'_{bc} - \bar{f}'_{bc} - 2\bar{f}'_t)/\Delta \end{aligned} \tag{8}$$

where

$$\Delta = (2\bar{f}'_{bc} - 1)(2\bar{f}'_{bc} + \bar{f}'_t)(1 + \bar{f}'_t) \tag{9}$$

Substituting Eq. (6) into Eq. (5) and rewriting Eq. (5), the Bresler–Pister yield function in terms of stress invariant can be obtained, and it is given as

$$f = \left(\frac{c}{9f'^2_c}\right)I_1^2 - \left(\frac{b}{3f'_c}\right)I_1 - \left(\frac{\sqrt{J_2}}{\sqrt{3}f'_c}\right)\sqrt{J_2} - a. \tag{10}$$

3. Plastic material matrix for concrete based on Bresler–Pister criterion

In the plasticity theory, total strain can be assumed to be the sum of the elastic strain and plastic strain as given in Eq. (11), and stress increment, $d\sigma_{ij}$, for strain increment, $d\varepsilon_{ij}$, is given in Eq. (12) [15].

$$d\varepsilon_{ij} = d\varepsilon^e_{ij} + d\varepsilon^p_{ij} \tag{11}$$

$$d\sigma_{ij} = D^{ep}_{ijkl}d\varepsilon_{ij} \tag{12}$$

where D^{ep}_{ijkl} is elastic–plastic material matrix. In the case of associated flow rule the general form of this matrix is given as,

$$D^{ep}_{ijkl} = D_{ijkl} + D^p_{ijkl} \tag{13}$$

Table 1
Test data for Bresler–Pister Criterion.

Test	σ_{oct}/f'_c	τ_{oct}/f'_c
$\sigma_1 = f'_t$	$\frac{1}{3}\bar{f}'_t$	$\frac{\sqrt{2}}{3}\bar{f}'_t$
$\sigma_3 = -f'_c$	$-\frac{1}{3}$	$\frac{\sqrt{2}}{3}$
$\sigma_2 = \sigma_3 = -f'_{bc}$	$-\frac{2}{3}\bar{f}'_{bc}$	$\frac{\sqrt{2}}{3}\bar{f}'_{bc}$