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Influence of continuum damage mechanics on the Bree's diagram of a closed end tube

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ABSTRACT

This paper extends the Bree's cylinder behaviors, which is subjected to the constant internal pressure and cyclic temperature gradient loadings, with considering continuum damage mechanics coupled with non-linear kinematic hardening model. The Bree's biaxial stress model is modified using the unified damage and the Armstrong–Frederick nonlinear kinematic hardening models. With the help of the return mapping algorithm, the incremental plastic strain in axial and tangential directions is obtained. Continuum damage mechanics approach can be used to extend the Bree's diagram to the damaging structures and reduce the plastic shakedown domain. Kinematic hardening behavior was considered in the material model which shifts the ratcheting zone. The role of the material constants in the Bree's diagram is also discussed.

1. Introduction

The need for a suitable constitutive model to predict the shakedown or cyclic failure (ratcheting) of structures under cyclic loading is increasing in many industries, and in order to better obtain the structures behaviors, many researchers tried to develop improved constitutive models [1]. Although, the proposed models simulate well the uniaxial ratcheting responses, there are other factors that influence the biaxial stress cyclic loading. Shakedown loads and different behaviors of structures were also studied by many authors with the plasticity and cyclic plasticity models [2]. In particular, the two-bar problem and the Bree's cylinder were studied under different loading conditions and materials behaviors. Parkes [3] studied thermal ratcheting in an aircraft wing resulting from the cyclic thermal stresses superimposed on the normal wing loads. Miller [4] showed that the material strain hardening reduces considerably the strains due to ratcheting in the two-bar structure. Jiang and Leckie [5] presented a method for direct determination of the steady solutions in shakedown analysis with application to the two-bar problem. Bree [6] analyzed the elastic–plastic behavior of a thin cylindrical tube subjected to constant internal pressure and cyclic temperature gradient across the tube thickness. A simple one-dimensional model, a linear temperature drop distribution across the cylinder thickness and an elastic–perfectly-plastic material model were assumed in his analysis. Later, he used a biaxial stress model and obtained a more complete interaction diagram (Bree's diagram) for a closed tube [7]. In that study the material was assumed to be perfect plastic. The Poisson's effect was also neglected. The interaction diagram proposed by Bree received adequate investigation by a number of researchers [8,9]. His one-dimensional diagram is a part of ASME boiler and pressure vessel code [8] (Fig. 1). Mulcahy [9] improved the Bree's analysis with incorporating a linear kinematic hardening model in the analysis of a beam element.

Classical shakedown theory of Melan–Koiter for elastic–perfectly-plastic bodies has been well established in the literature [10]. Melan procedure for the shakedown theorem can also be extended to encompass the linear kinematic hardening, but one encounters mathematical difficulty in treating more general cases, and the procedure does not apply straightforwardly for the case of Prager's linear kinematic hardening. It seems that some additional assumptions as well as further mathematical tricks are needed to deal with the general kinematic hardening. Recently, Abdalla et al. [11] proposed a simple shakedown method with perfect plasticity material behavior to study the Bree's cylinder problem and the 90° pipes bending.

Very limited work has been done on damage related to the interaction diagram. The shakedown theory has been extended to include hardening and damage by Hachemi and Weichert [12] and Druyanov and Roman [13]. However, the extensions should be made often at the expense of losing certain fine features of the classical plasticity theory and shakedown theorems. Without the theorems in Melan–Koiter sense, which are valid only with certain restrictions, generally in practice one has to implement numerical incremental analysis to check for shakedown of a

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Nomenclature

A	virgin surface	ΔT_x	temperature difference in each radius
$\tilde{A}$	resistant effective surface	x	coordinate
C	NLKH model's constant	$\boldsymbol{X}$	back stress tensor
D_c	interatomic decohesion damage parameter	$\boldsymbol{X'}$	deviatoric back stress tensor
E	elastic modulus	Y	associate thermodynamics damage variable tensor
$\tilde{E}$	effective elastic modulus	α	expansion coefficient
e_z	nondimensional axial strain	γ	NLKH model's constant
e_θ	nondimensional tangential strain	$d\lambda$	incremental plastic multiplier
f	yield function	η	nondimensional thickness
F_ϖ	dissipative potential function	$\boldsymbol{\varepsilon^p}$	plastic strain tensor
K_1	mean tangential strain	$\boldsymbol{\varepsilon^T}$	thermal strain tensor
K_2	mean axial strain	ε_D^p	plastic damage threshold strain
n_{ij}	outward normal to the yield surface	$\boldsymbol{\sigma}$	stress tensor
P	internal pressure	$\tilde{\boldsymbol{\sigma}}$	effective stress tensor
$\dot{p}$	equivalent plastic strain rate	$\boldsymbol{\sigma}'$	deviatoric stress tensor
Δp	equivalent plastic strain increment	σ_{eq}	equivalent von-Mises stress
q	material damage constant	σ_H	hydrostatic stress
Q	material damage constant	σ_P	nondimensional mechanical stress parameter
r	mean radius	σ_y	yield stress
R_υ	triaxiality function	τ	nondimensional temperature difference
S_z	nondimensional axial stress	$\Delta\tau$	nondimensional temperature difference increment
S_θ	nondimensional tangential stress	υ	Poisson's ratio
t	thin wall thickness	ϖ	damage parameter
T	temperature		
ΔT	temperature difference between the inner and outer surfaces		

structure under specific loading histories. Recently, Nayebi and El Abdi [14] and Kang et al. [15] used continuum damage mechanics (CDM) to predict the material behavior, including ratcheting and shakedown in 1-D analysis.

In this research, the effect of continuum damage mechanics on the Bree's diagram of a thin cylinder structure under specific loading histories is studied. Nonlinear kinematic hardening (NLKH) theory is coupled with continuum damage mechanics in order to model the behavior of the Bree's cylinder. A unified damage mechanics model, which is also appropriate for low cyclic loading, is used. During each loading, the damage analysis is performed. An iterative method is used to analyze the cylinder under the cyclic thermal and constant mechanical loads. The model extends Bree's 2-D diagram to incorporate damage effects. The proposed method can also be applied to other structures subjected to cyclic thermal and constant mechanical loadings. The results illustrate the influence of material damage on the behaviors of structures under cyclic loading in comparison with the confirmed results on undamaged structures.

2. Constitutive behavior relations

2.1. Continuum damage mechanics

According to the applied theory of damage mechanics, microscopic change in a material element of surface A develops into macroscopic defect as a result of loading. In the damaged state, the new area is denoted by $\tilde{A}$ (Fig. 2), from which the isotropic damage variable ϖ is defined as [16]

$$\varpi = \frac{A - \tilde{A}}{A}, \tag{1}$$

where ϖ may be considered as an internal state variable characterizing the irreversible deterioration of a material in the thermodynamic sense. Following this theory, the behavior of a damaged material can then be represented by the constitutive equations of the virgin material where the usual stress tensor, σ, is replaced by the effective stress $\tilde{\boldsymbol{\sigma}}$ defined by

$$\tilde{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}}{1 - \varpi}, \tag{2}$$

where the value $\varpi = 0$ corresponds to the undamaged state, $\varpi \in (0, D_c)$ corresponds to a partly damaged state, and $\varpi = D_c$ defines the element state rupture by interatomic decohesion ($D_c \in [0, 1]$). In the sequel, superposed tilde indicates quantities related to the damaged state of the material.

From a physical point of view, the material degradation involves the initiation, growth and coalescence of micro-cracks or micro-voids generally induced by large plastic strains. This phenomenon is called ductile plastic damage and leads to plastic (ductile) fracture. Many observations and experiments indicated that the damage is also governed by the plastic strain which is introduced into the model through the plastic multiplier $\dot{\lambda}$, as [16]

$$\dot{\varpi} = \dot{\lambda}\frac{\partial F_\varpi}{\partial Y} \quad \text{if } \varepsilon^p \succ \varepsilon_D^p, \tag{3}$$

where $\dot{\lambda}$ is calculated from the constitutive equations of plasticity coupled with the damage deduced from the dissipative potential function, F_ϖ. Y is the associate variable of the damage rate, $\dot{\varpi}$, and ε_D^p is the plastic damage threshold strain. Also, many experimental results indicated that F_ω must be a nonlinear function of Y [16]

$$F_\varpi = \frac{Q}{(q+1)(1-\varpi)}\left(\frac{Y}{Q}\right)^{q+1} \tag{4}$$

from which Eq. (3) reduces to

$$\dot{\varpi} = \left(\frac{Y}{Q}\right)^q \dot{p} \tag{5}$$

where Q and q are material parameters and $\dot{p}$ is the equivalent plastic strain rate. According to Lemaitre and Desmorat [16]