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Accurate determination of coupling effects on free edge interlaminar stresses in piezoelectric laminated plates

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ABSTRACT

A layerwise laminate theory is used to investigate analytically the electromechanical coupling effects on the interlaminar stresses and the electric field strengths near the free edges of laminated plates with piezoelectric material properties. The angle-ply and cross-ply piezoelectric laminated plates with finite width subjected to uniform axial strain are considered and full three-dimensional stresses and the electric field strengths in the interior and boundary layer regions are calculated. Equilibrium equations along with the appropriate boundary conditions are obtained by using the principle of minimum total potential energy. The analytical solutions are developed and through the comparison of the present results with those available in the literature, the accuracy of the theory is verified. It is found that the interlaminar stresses and electric field strengths at the free edges are significantly higher in the coupled case for cross-ply laminates, whereas the coupling effect for symmetric angle-ply laminates is of minor significance.

1. Introduction

The field of smart materials and structures is interdisciplinary between science and technology and combines the knowledge of physics, mathematics, chemistry, and computer science, with material, electrical, and mechanical engineering. Smart structures can help us to control the environment better and to increase the energy efficiency of devices. These structures are usually systems containing multifunctional components that can perform sensing, control, and actuation. Example of such smart structures include piezoelectric transducers that are used in many applications such as structural vibration control, precision positioning, aerospace, and more recently they have been critical in advancing research in nanotechnology. Piezoelectric devices have several advantages over other sensing mechanisms. Since this sensor generates its own voltage, it does not require power of operation. Therefore in applications where power consumption is a significant constraint, piezoelectric devices can be used. Piezoelectric transducers are available in many forms and shapes. The most widely used piezoelectric transducers are in the form of thin sheets that can be bonded to or embedded in composite structures. As actuators they are mainly used to generate moment in flexible structures, while as sensors they are used to measure strain. Piezoelectric actuators are also available in the form of stacks, where many layers of materials and electrodes are assembled together. These stacks generate large force but small displacement in the direction normal to the top and bottom surfaces [1,2].

Piezoelectric devices are often made from multi-layered thin films of dissimilar materials, which bring about a number of interfaces and/or edges. Interfaces are intrinsic to these materials and have a special significance because they may just affect the structural performance of such materials and systems. However, they are susceptible to delaminate in the processing and in service. Especially, the delamination crack usually initiates at the free edge of thin films due to the stress concentration originated from the mismatch of deformation. In practice, delamination along the interface is found one of the major failure mechanisms occurred in various thin film/substrate systems. Another challenge concerning the structural integrity and reliability comes from the brittle nature of piezoelectric ceramics. Therefore, it is of critical importance to evaluate the interface strength between the thin films on a substrate.

Interlayer stresses and the effect of their concentration on mechanical performance have been investigated by many researchers and scientists. Early studies were focused mainly on laminated composite structures. Comprehensive reviews on developments in this field can be found in [3–5]. During the past two decades, the interlayer stresses and failure mechanism of adaptive structures have caused increasingly attention of practical engineers and professional researchers. Due to the difficulty in obtaining theoretical solution for such a complex problem, the finite element method (FEM) is usually employed to model and simulate stress transfer and failure process of interface between layers of

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adaptive structures [6–10]. Yang et al. [11] studied interfacial mechanical behavior of laminated beams, consisting of two piezoelectric facial sheets and an elastic core. The study was based on coupled multi-field finite element formulation and they investigated the interlayer stresses and their concentrations near free edge in laminated beams of piezoelectric and elastic materials.

Artel and Becker [12] employed eight-nodded isoparametric displacement based volume elements with refined free edge modeling and investigated piezoelectric coupling effects in symmetric cross-ply and angle-ply laminates. They found that for cross-ply laminates, some interlaminar stress components near the free edge become singular and in the coupled analysis case are usually of higher magnitude than in the uncoupled analysis. The static interaction between a laminate and distributed piezoelectric actuators was considered by Mannini and Gaudenzi [13]. They used a multilayer higher-order finite element to model a laminated composite plate. The effectiveness of the method was evaluated referring to some typical cases and a good agreement with similar other results was obtained. Erturk and Tekinalp [14] investigated multilayer plates with piezoelectric layers considering the deformation of adhesive layers. They showed that although deformed shapes are quite similar, local stresses in adhesive layers differ especially near delaminated region. Also, Chrysochoidis and Saravanos [15] formulated a coupled linear layerwise laminate theory and a beam finite element for analyzing delaminated composite beams with piezoactuators and sensors.

The numerical methods illustrate the strong effect of delamination on the sensor voltage, on through the thickness displacement, and on the stress field. Although the finite element method is a powerful tool for the numerical analysis of smart structures and systems, it does not effectively delineate the interrelationship among the various piezoelectric and elastic constants, namely, their coupling effects. Generally, FEM is not highly desirable for parametric studies. Although the theory of linear piezoelectricity is well developed [16–21], there are few analytical solutions including the free edge effects in piezoelectric laminated plates. To this end, various kinds of reduced models have been recently developed for piezoelectric plates. Generally, the reduced models are based on some assumptions, if the assumptions are applied to the entire laminate, they are equivalent single-layer models and if the assumptions are applied to each layer within the laminate, they are referred as layerwise models. Izadi Najafabadi and Tahani [22] determined analytically the interlaminar stresses at the free edges of long symmetric cross-ply laminates with piezoelectric material properties by using an equivalent single-layer model. Also Tahani and Mirzababaee [23] presented coupled and uncoupled analyses of free edge effect in piezoelectric laminates using a higher-order shear-thickness deformation theory. To the extent of the author's knowledge, no other work has been reported for analytical study of the coupling effects on free edge interlaminar stresses of piezoelectric laminated plates.

In addition to the theoretical models, several investigators also used various nondestructive experimental techniques and methods to detect delamination and measure interlaminar stresses in laminated composite structures by using piezoelectric sensors [24–27].

In this paper, by using a layerwise displacement theory an analytical solution is developed to determine the interlaminar stresses and the electric field strengths in the vicinity of free edges of long cross-ply and balanced symmetric angle-ply piezoelectric laminated plates subjected to uniform constant axial strain. In this regard, two cross-ply and one angle-ply laminates are analyzed to show the influence of piezoelectric coupling on the interlaminar stresses and electric field strengths near the free edges. Then the capability of the present approach is characterized by illustrating the local variation of electromechanical variables such as stresses and electric potential at the layer interfaces and through the thicknesses of the piezoelectric laminated plates.

2. Mathematical formulation

As it is pointed out, it is intended here to accurately determine the interlaminar stresses and electric field strengths in cross-ply and balanced symmetric angle-ply piezoelectric laminated plates subjected to uniform constant axial strain ε_0. The geometry of the laminate is shown in Fig. 1. The formulation is restricted to linear piezoelectric material behavior and small strain and displacements. In the present study, because the stress resultants are being imposed at the interface corners instead of stress components, the symmetric stress tensor of linear elasticity is used. By using the principle of minimum total potential energy, equilibrium equations are obtained in terms of stress and electric flux resultants. Next, the displacement equilibrium equations are obtained. Finally, the set of governing equilibrium equations are solved analytically by using the state space approach.

2.1. Layerwise concept

The layerwise theory is in fact a generalization of the higher-order shear deformation theories (HSDTs). The layerwise theory has an arbitrary order in general and thereby each layer has its own polynomial function. The layerwise theory is similar to a "separation of variables" technique in which the *z* or thickness dimension is decoupled from the *x*–*y* dimensions. The layerwise theory used in the present study assumes separate displacement field expansions within each material layer that exhibits only C^0-continuity through the laminate thickness. Therefore, the resulting strain field is kinematically correct, especially when the number of numerical

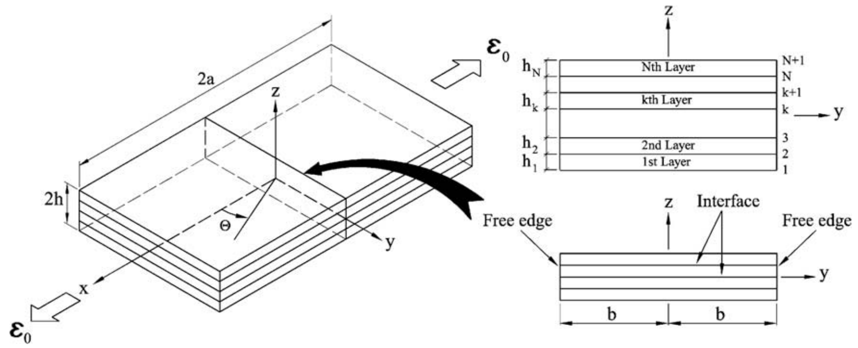

Fig. 1. Geometry and coordinate system of a piezoelectric laminated plate.