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Controlling thermal deformation by using composite materials having variable fiber volume fraction

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ABSTRACT

In application, many thin structural components such as beams, plates and shells experience a through-thickness temperature variation. This temperature variation can produce both an in-plane expansion and an out-of-plane (bending) curvature. Given that these thin components interact with or connect to other components, we often wish to minimize the thermal deformation or match the thermal deformation of another component. This is accomplished by using a composite whose fibers have a negative axial thermal expansion coefficient. By varying the fiber volume fraction within a symmetric laminated beam to create a functionally graded material (FGM), certain thermal deformations can be controlled or tailored. Specifically, a beam can be designed which does not curve under a steady-state through-thickness temperature variation. Continuous gradation of the fiber volume fraction in the FGM layer is modelled in the form of a mth power polynomial of the coordinate axis in thickness direction of the beam. The beam results are independent of the actual temperature values, within the limitations of steady-state heat transfer and constant material properties. The influence of volume fiber fraction distributions are studied to match or eliminate an in-plane expansion coefficient, or to match a desired axial stiffness. Combining two fiber types to create a hybrid FGM can offer desirable increase in axial and bending stiffness while still retaining the useful thermal deformation behavior.

1. Introduction

Because of their high strength-to-weight and stiffness-to-weight ratios, composite materials are being increasingly used in structural applications such as the aeronautics, astronautics, national defences and nuclear energy and civil engineering. In conventional configurations, flat beams and plates are made of plies, the fibers within each ply being parallel and uniformly spaced. However, it is possible that significant increases in structural efficiency may be obtained by varying the fiber spacing packing them closely together in regions where great stiffness is needed, but less densely in other regions. Meftah et al. [1] develop a finite element model for static and free vibration analysis of reinforced concrete (RC) shear walls structures strengthened with thin composite plates having variable fibers spacing. Benatta et al. [2] studied the static behavior of symmetric functionally graded beams under three-point bending using displacement field based on higher order shear deformation theory. In their study, the functionally graded material (FGM) is created by varying the fiber volume fraction within a symmetric laminated beam. Meftah et al. [3] investigated the seismic response of the RC coupled shear walls structures strengthened with CFRP laminates having variable fibers spacing. Kubiak [4] studied the dynamic response of a thin-walled plate with varying widthwise material properties subjected to in-plane pulse loading of rectangular shape.

When a structure is subjected to a non-uniform temperature field, it normally reacts by producing deformations composed of (in-plane) expansion and (out-of-plane) bending. These deformations are usually undesirable since they distort the structure and cause stresses when parts expand unequally. Wetherhold and Wang [5] and Wetherhold et al. [6] have studied how we can eliminate or control these thermal deformations and we envisage extending their works in the present paper to study the case of composite beams having variable fiber distributions through the thickness of beams according to mth power polynomial law.

With the availability of materials such as graphite and Kevlar fibers which possess a negative axial thermal expansion coefficient and high stiffness, a composite may be made which exhibits a negative axial coefficient of thermal expansion (CTE). If we desire a structure with zero CTE in the plane, we can do this by combining layers with positive CTE with layers having negative CTE. We can also fabricate a composite whose fiber volume fraction (V_f) equals the critical value (V_c) so that the CTE is everywhere zero (although the resulting composite may have low V_f and may thus have low stiffness). Zero CTE ensures dimensional stability and minimal

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thermal mismatch stresses amongst similar parts of a structure subject to a uniform temperature field. If we join zero CTE components with non-zero CTE components, however, there will be mismatch stresses and distortion even for a uniform temperature.

For the case of a non-uniform temperature field, the problem becomes more complex. In actual application, the structure usually experiences a temperature variation through its thickness as one side is exposed to sunlight, an impinging gas or liquid, or a heat source. The zero in-plane CTE laminated composite structures do not provide protection against this temperature field. If they are in laminated form, they will bend, and the bending deformation can be considerable. A beam with uniform volume fraction, $V_f = V_c$ (the critical V_f value), can provide protection against bending, but will have an in-plane CTE of zero (may not match neighboring structures) and may be insufficiently stiff because of low V_f.

Wetherhold and Wang [5] and Wetherhold et al. [6] have proposed design methods to control both thermal deformation and mismatches in thermal deformation. One method is to bond dissimilar composite or composite and metal layers together [5]. This creates potential manufacturing difficulties concerning the layer interfaces. If it is not possible to co-cure the beam, then a separate surface preparation and bonding step will be needed. The other method is to use functionally graded materials (FGM) in the form of composites whose volume fraction varies through the thickness (according to linear or parabolic variation) to avoid the difficulties of bonding dissimilar layers together [6]. In the present paper, we use FGM beams with variation of volume fraction of fibers based on mth power polynomial law. The effect of power-law exponent on thermal deformation is also commented.

2. Mathematical model

2.1. Continuous variation of fiber volume fraction

Consider a symmetric beam of thickness $2h$ (Fig. 1). The fiber volume fraction, V_f vary continuously through the thickness of beam (in z-direction) as follows:

$$V_f(z) = V_2 + (V_1 - V_2)\left(\frac{|z|}{h}\right)^m \tag{1}$$

where $V_1 = V_f(h)$ and $V_2 = V_f(0)$. z = distance from mid-surface and m is power-law index, a positive real number.

The stiffness and thermal expansion coefficient for a uniaxial reinforcement are given by

$$E(z) = E_f V_f(z) + E_m(1 - V_f(z)) \tag{2a}$$

$$\boldsymbol{\alpha}(z) = \frac{\boldsymbol{\alpha}_f E_f V_f(z) + \boldsymbol{\alpha}_m E_m(1 - V_f(z))}{E(z)} \tag{2b}$$

where subscripts f and m refer to fiber and matrix, respectively. Eq. (2) is simple rule-of-mixtures approximations which are accurate for unidirectional, continuous-fiber composites [7].

To describe the deformation of the symmetric beam, we evaluate the thermal and mechanical force and moment resultants and relate them, through laminate stiffnesses, to the bending curvature and in-plane strain. This also allows us to calculate the CTE of the beam. The mid-plane strain, $\boldsymbol{\varepsilon}_x^0$, and the bending curvature, k_x, are related to applied mechanical force, N_x, and moment, M_x, resultants and thermal force, N_x^T, and moment, M_x^T, resultants by

$$N_x = A\boldsymbol{\varepsilon}_x^0 - N_x^T \tag{3}$$

$$M_x = Dk_x - M_x^T \tag{4}$$

The beam has in-plane stiffness, A, and out-of-plane or bending stiffness, D, given by

$$A = \int_{-h}^{h} E(z)\,dz > 0 \tag{5a}$$

$$D = \int_{-h}^{h} E(z)z^2\,dz > 0 \tag{5b}$$

The mechanical and thermal force and moment resultants are defined by

$$N_x = \int_{-h}^{h} \sigma_x\,dx \tag{6a}$$

$$N_x^T = \int_{-h}^{h} E(z)\boldsymbol{\alpha}(z)T(z)\,dz \tag{6b}$$

$$M_x = \int_{-h}^{h} \sigma_x z\,dz \tag{7a}$$

$$M_x^T = \int_{-h}^{h} E(z)\boldsymbol{\alpha}(z)T(z)z\,dz \tag{7b}$$

Consider the case of zero applied mechanical loading ($N_x = M_x = 0$) to obtain the mid-surface thermally induced strain, $\boldsymbol{\varepsilon}_x^0$, and the thermal curvature, k_x. Eqs. (3) and (4) will reduce and can be inverted to give

$$\boldsymbol{\varepsilon}_x^0 = \frac{1}{A}N_x^T \tag{8}$$

$$k_x = \frac{1}{D}M_x^T \tag{9}$$

The effective laminate in-plane CTE ($\boldsymbol{\alpha}_x$) can be found by simplifying Eq. (8) for the case of a uniform temperature field:

$$\boldsymbol{\alpha}_x \equiv \frac{\boldsymbol{\varepsilon}_x^0}{\Delta T} = \frac{1}{A}\int_{-h}^{h} E(z)\boldsymbol{\alpha}(z)\,dz \tag{10}$$

The temperature profile, $T(z)$, is linear through the thickness if the transverse thermal conductivity, k, is constant. If k were a strong function of V_f, the temperature profile would deviate from linearity and the beam design would have to be 'tuned' for a specific temperature profile. In fact, k is a weak function of $V_f(z)$ for a polymer matrix with typical fibers [8]. It has been shown that the importance of k is distinctly secondary to that of E and $\boldsymbol{\alpha}$ [9], and k is assumed constant in this paper. The temperature profile is thus

$$T(z) = T_1 + \frac{(T_2 - T_1)}{2h}(z + h) \tag{11}$$

where (T_1, T_2) is the temperature with $T_1 = T(z = -h)$ and $T_2 = T(z = h)$.

The properties of the fibers and the epoxy matrix used in this paper are given in Table 1. For each fiber, there is a critical volume fraction, V_c, at which the longitudinal CTE is zero (see Eq. (2b)). A

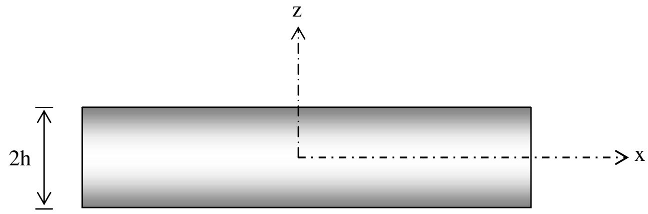

Fig. 1. Symmetric functionally graded beam.

Table 1
Fiber and matrix properties.

	AS Gr	Kevlar	P100 Gr	Epoxy
E (GPa)	277.67	137.76	772.0	3.5
$\boldsymbol{\alpha}$ (10^{-6}/C)	−0.969	−5.49	−1.40	65
V_c (%)	50.8	23.1	17.4	–