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Thermal buckling load optimization of angle-ply laminated cylindrical shells

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ABSTRACT

In this study, thermal buckling load optimization of symmetrically laminated cylindrical shells subjected to uniformly distributed temperature load is investigated. The objective function is to maximize the critical temperature capacity of laminated shells and the fibre orientation is considered as design variable. The first-order shear deformation theory is used to study thermal buckling response of the laminates. The modified feasible direction method is used as optimization routine. For this purpose, a program based on FORTRAN is used for the optimization of shells. Finally, the effects of number of layers, length-to-radius ratio and boundary conditions on the optimum results are investigated.

1. Introduction

The advances in composite technology have lead to the application of elevated-temperature composite structural elements like cylindrical shells in the design of more and more sophisticated futuristic structural systems such as supersonic and hypersonic aircraft, rockets, satellites, nuclear components, etc. However, one of the problems is associated with the determination of the buckling behavior of laminated shells under such environmental conditions, which is essential for a better understanding and exploitation of their load-carrying capacity.

The thermal buckling studies of the laminated composite shells have received limited attention in the literature compared to those of isotropic shells and laminated plates. Matsunaga [1] presented a two-dimensional global higher-order deformation theory for the evaluation of the critical temperatures in the cross-ply laminated composite shallow shells subjected to the thermal load. Lee et al. [2] performed numerical simulation analyzes of the thermal buckling behavior of the laminated composite shells with embedded shape memory alloy (SMA) wires to investigate the effect of the embedded SMA wires on the characteristics of the thermal buckling. Patel et al. [3] studied thermoelastic buckling characteristics of the angle-ply laminated elliptical cylindrical shells subjected to the uniform temperature rise to highlight the influences of the non-circularity and ply-angle on the critical temperature parameter and buckling mode shapes. Patel et al. [4] studied thermoelastic stability characteristics of the cross-ply laminated oval cylindrical/conical shells subjected to the uniform temperature rise through nonlinear static analysis employing the finite element approach. The study was carried out to highlight the influences of the non-circularity parameter, number of layers, material properties and semi-cone angle on the nonlinear thermoelastic response/stability characteristics of the laminated oval cylindrical/conical shells. Shen and Li [5] investigated the effects of the local geometric imperfections on the buckling and post-buckling of the shear deformable laminated cylindrical shells subjected to the combined axial compression and uniform temperature loads. Wang et al. [6] investigated the non-linear local thermal buckling of an elliptic, triangular and lemniscates local delamination near the surface of the laminated cylindrical shells. Shen [7] presented a post-buckling analysis of a stiffened laminated cylindrical shell subjected to the combined loads of external pressure and a uniform temperature rise. Numerical examples were presented for the performance of perfect and imperfect, stiffened and unstiffened cross-ply laminated cylindrical shells. Gotsis and Guptill [8] presented parametric studies to assess the effects of the various parameters on the buckling behavior of the angle-ply laminated thin shells in a hot environment. Parametric studies were performed to examine the effects of the cylinder length and thickness, internal hydrostatic pressure, different ply thicknesses, different temperature profiles through the thickness of the structure, different layup configurations and fiber volume fractions on the critical buckling load. Chang and Chiu [9] analyzed thermal buckling analysis of simply supported antisymmetric angle-ply laminated cylindrical shells subjected to a uniform temperature rise by a finite element method based on the higher-order displacement functions. Thangaratnam et al. [10] investigated linear buckling analysis of the laminated composite cylindrical and conical shells under thermal load. Critical temperatures were presented for various cases of cross-ply and angle-ply laminated shells. The effects of radius/thickness ratio, number of layers, ratio of coefficients of thermal expansion and the angle of fiber orientation on the critical buckling load were studied. Eslami and Javaheri [11] studied buckling of the

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specially orthotropic laminated composite cylindrical shells under mechanical and thermal loads. Birman and Bert [12] investigated the effects of the temperature on the buckling and post-buckling behavior of the reinforced and unstiffened composite plates or cylindrical shells. Wu and Chiu [13] presented an asymptotic theory for the thermoelastic buckling analysis of the laminated composite conical shells subjected to a uniform temperature change. The critical thermal loads of simply supported, cross-ply conical shells were studied to demonstrate the performance of the present asymptotic theory.

On the other hand, thermal buckling load optimization of laminated cylindrical shells has not been investigated by authors until now. Therefore, in this study thermal buckling load optimization of laminated angle-ply cylindrical shells subjected to uniformly distributed temperature load is investigated to fill this gap. The objective function is to maximize the critical temperature capacity of laminated shells and the fibre orientation is considered as design variable. The first-order shear deformation theory is used to study thermal buckling response of the laminated cylindrical shells. The modified feasible direction method (MFDM) is used as optimization routine. For this purpose, a program based on FORTRAN is used for the optimization of shells. Finally, the effects of number of layers, length-to-radius ratio and boundary conditions on the optimum results are investigated and the results are compared.

2. Basic equations

Consider a laminated cylindrical shell of length L, radius R and thickness H with both ends simply supported (Fig. 1). The shell has a symmetric layup consisting of N layers of equal thickness. The structure is referenced in an orthogonal coordinate system (x, θ, z), where x, θ and z are the longitudinal, circumferential and radial directions, respectively.

Based on the first-order shear deformation theory (FSDT), in-plane and transverse displacements in the kth layer are assumed in the following form:

$$\begin{aligned} u^k &= u_0(x,\theta,t) + z\phi_x(x,\theta,t) \\ v^k &= v_0(x,\theta,t) + z\phi_\theta(x,\theta,t) \\ w^k &= w_0(x,\theta,t) \end{aligned} \tag{1}$$

Here, u_0, v_0, w_0 reply for the displacements of a point in the mid or reference surface in the x, θ and z directions, respectively; φ_x and φ_θ are the rotations of normal to the mid-surface about θ and x axes, respectively.

The stress resultants N_x, N_θ, $N_{x\theta}, N_{\theta x}$, moment resultants $M_x, M_\theta, M_{x\theta}, M_{\theta x}$ and the transverse shear forces Q_x, Q_θ in the laminated shell are

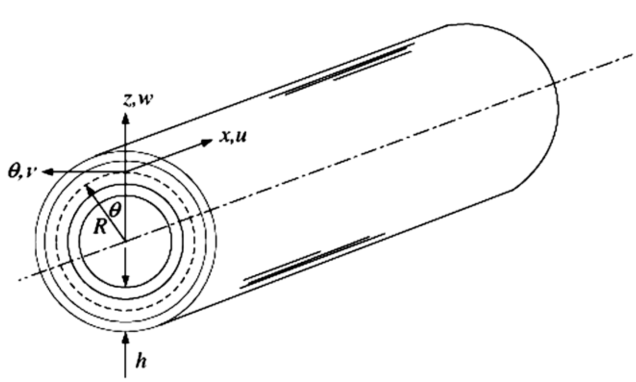

Fig. 1. Geometry of a laminated cylindrical shell.

$$\begin{aligned} \{N_x, N_\theta, N_{x\theta}, N_{\theta x}\} &= \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \{\sigma_x(1+z/R), \sigma_\theta, \sigma_{x\theta}(1+z/R), \sigma_x\}\mathrm{d}z \\ \{M_x, M_\theta, M_{x\theta}, M_{\theta x}\} &= \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \{\sigma_x(1+z/R), \sigma_\theta, \sigma_{x\theta}(1+z/R), \sigma_x\}z\mathrm{d}z \end{aligned} \tag{2}$$

$$\{Q_x, Q_\theta\} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \{\sigma_{xz}(1+z/R), \sigma_{\theta z}\}\mathrm{d}z$$

The constitutive relations for a laminated cylindrical shell accounting for the thermal effects can be written as

$$\begin{pmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{x\theta} \end{pmatrix}_{(k)} = \begin{pmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{pmatrix}_{(k)} \begin{pmatrix} \varepsilon_x - \alpha_x\Delta T \\ \varepsilon_\theta - \alpha_\theta\Delta T \\ \varepsilon_{x\theta} - \alpha_{x\theta}\Delta T \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} \tau_{\theta z} \\ \tau_{xz} \end{pmatrix}_{(k)} = \begin{pmatrix} \bar{Q}_{44} & \bar{Q}_{45} \\ \bar{Q}_{45} & \bar{Q}_{55} \end{pmatrix}_{(k)} \begin{pmatrix} \gamma_{\theta z} \\ \gamma_{xz} \end{pmatrix} \tag{4}$$

where $\bar{Q}_{ij}$ is the transformed reduced stiffnesses, α_x, α_θ, $\alpha_{x\theta}$ are the coefficients of thermal expansion and ΔT is the uniform constant temperature difference.

The strain components of the shell is expressed as follows:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{\theta z} \\ \gamma_{xz} \\ \varepsilon_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{R\partial\theta} + \frac{w}{R} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{R\partial\theta} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} + \frac{\partial u}{R\partial\theta} \end{Bmatrix} \tag{5}$$

By performing the similar procedure in Eq. (1), the strain components are obtained as the forms:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{\theta z} \\ \gamma_{xz} \\ \varepsilon_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_\theta^0 \\ \gamma_{\theta z}^0 \\ \gamma_{xz}^0 \\ \varepsilon_{x\theta}^0 \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_\theta^1 \\ \gamma_{\theta z}^1 \\ \gamma_{xz}^1 \\ \varepsilon_{x\theta}^1 \end{Bmatrix} \tag{6}$$

where

$$\{\varepsilon^0\} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_\theta^0 \\ \gamma_{\theta z}^0 \\ \gamma_{xz}^0 \\ \varepsilon_{x\theta}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{R\partial\theta} + \frac{w_0}{R} \\ \frac{\partial v_0}{\partial z} + \frac{\partial w_0}{R\partial\theta} \\ \frac{\partial u_0}{\partial z} + \frac{\partial w_0}{\partial x} \\ \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{R\partial\theta} \end{Bmatrix}, \quad \{\varepsilon^1\} \begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_\theta^1 \\ \gamma_{\theta z}^1 \\ \gamma_{xz}^1 \\ \varepsilon_{x\theta}^1 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial^2 u}{\partial x\partial z} \\ \frac{\partial^2 v}{R\partial\theta\partial z} \\ 0 \\ 0 \\ \frac{\partial^2 v}{\partial x\partial z} + \frac{\partial^2 u}{R\partial\theta\partial z} \end{Bmatrix} \tag{7}$$

The general form of Eq. (2) can be written as:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A_{ij} & 0 \\ 0 & D_{ij} \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \varepsilon^1 \end{bmatrix}, \begin{bmatrix} Q_x \\ Q_\theta \end{bmatrix} = K \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{bmatrix} \gamma_{xz}^0 \\ \gamma_{x\theta}^0 \end{bmatrix} \tag{8}$$

where A_{ij} and D_{ij} are extensional and bending stiffnesses, respectively, which are defined in terms of the lamina stiffness $\bar{Q}_{ij}$ as

$$\{A_{ij}, D_{ij}\} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{Q}_{ij}(1, z^2)\mathrm{d}z \tag{9}$$

where z_k and z_{k+1} denote the distances from the shell reference surface to the outer and inner surfaces of the kth layer and K is the shear correction factor. In this study, the shear correction factor is taken 5/6 [14].

The finite element used for the thermal buckling analysis of the laminated shell is Lagrange quadratic shell element, having nine