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Yield surfaces and micro-failure mechanism of block lattice truss materials

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ABSTRACT

The stiffness and the strength of block lattice truss materials were derived, as well as polyhedrical yield surfaces. Tension yield, compression yield and compression buckling of struts are the three main micro-failure mechanisms of the lattice materials. It is shown that when the relative density of the lattice is smaller than a critical value micro compression buckling of struts will dominate the macro failure mode of the material under macro shearing loading or even macro tensile loading. The lattice truss materials may be optimal designed according to their stacking mode of struts and the critical relative density.
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1. Introduction

Lattice truss materials are made up of stacks of struts interconnected at the nodes [1]. Lengths of struts are always at the level of centimeter. With the same topology as macro space truss structures, the nodes of lattice truss materials can be simplified as pin-jointed and the struts are mainly uniaxial stretched or compressed under macro loadings. To form this stretching dominated topology, not less than six stacks of struts are required to interconnect the periodic unit cells. Neglecting the bending effect, the lattice truss materials are much stronger and stiffer than traditional cellular materials. They are the most attractive lightweight structures [1] and possess attractive potential applications in the aerospace engineering and high speed transportation vehicles. Until now several types of lattice truss materials have been manufactured, including octet-truss materials [2], block lattice truss materials [3] and truss-core sandwiches with tetrahedral, pyramidal or diamond lattices [4–6]. Periodic unit cells of typical lattice structures as shown in Figs. 1 and 2 includes two classes. One special class are lattice truss materials made up of only six stacks of struts as shown in Fig. 1. With six stacks, the lattice truss materials are statically determinate. The stress of each strut can be acquired by the equilibrium equations of forces. Nodes of frameworks including octet-truss cells, octahedral cells and tetrahedral cells are all similarly situated. The connectivity of nodes is 12 and satisfied with the stretching dominated principle suggested by Deshpande et al. [1]. In diamond cells and pyramidal cells there are two types of nodes. One has the connectivity of 12 and the other has the connectivity of 6 or 8, which leads to anisotropic lattice truss materials. Another class of the lattice truss material is the block lattice truss material with seven stacks of struts as shown in Fig. 2. With more than six stacks, the stress distribution of each strut would be derived from the equilibrium equations and the deformation of the cells and the material is statically indeterminate.

With irregular micro structures, the mechanical properties of foams are hard to predict and the yield surfaces are always built with the help of experiments [7]. But for lattice truss materials with periodic and regular micro unit cells composed of straight struts, the mechanical properties are predictable and designable, which was proved by Deshpande et al. [2]. In this paper, the stiffness and the micro-failure mechanism of the statically indeterminate block lattice truss material would be studied.

2. Stiffness and micro-failure mechanism

Based on the elongation responses of struts under uniaxial forces, the relationship between the transformed stresses and strains can be derived to formulate the continuum model [8]. According to this method, the effective stiffness matrix $\mathbf{C}$ of the lattice material is given by [9]

$$\mathbf{C} = \mathbf{G}^{\mathrm{T}}\mathbf{E}\mathbf{G}, \tag{1}$$

where $\mathbf{G}$ is the coordinate transformation matrix from local micro strut to macro continuum, $\mathbf{E}$ is the effective tension stiffness of each strut. The vector of force $\mathbf{f}$ of each bar is related to the continuum stress $\boldsymbol{\sigma}$ by

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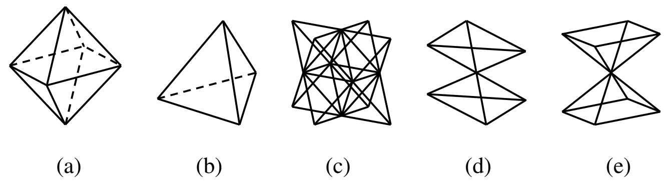

Fig. 1. Periodic unit cells of typical lattice truss materials: (a) octahedral cell; (b) tetrahedral cell; (c) octet-truss cell; (d) diamond cell; and (e) pyramidal cell.

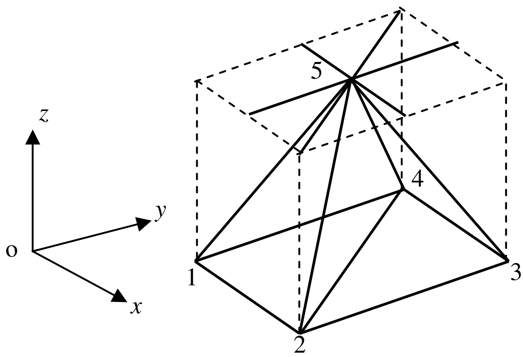

Fig. 2. Unit cell of the lattice block material.

$$\mathbf{f} = \mathbf{S}(\mathbf{G}^{\mathrm{T}})^{-1}\sigma \tag{2}$$

for 3D lattices with six stacks, or

$$\mathbf{f} = (\mathbf{SEGC}^{-1})\sigma \tag{3}$$

for 3D lattices with more than six stacks. Symbol $\mathbf{S}$ is the effective transection area distributed to each strut.

The block lattice truss material is made up of seven stacks of struts as shown in Fig. 2. Compared with other determinate lattice truss materials with six stacks of struts, such as octet-truss materials, the structure of the block lattice truss material is more complex and indeterminate. According to Eq. (1), the stiffness matrix of the lattice block material is

$$\mathbf{C} = \frac{\rho^* E_s}{6+\sqrt{2}} \begin{bmatrix} (5+\sqrt{2})/4 & (1+\sqrt{2})/4 & 1/2 & 0 & 0 & 0 \\ & (5+\sqrt{2})/4 & 1/2 & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & & & (1+\sqrt{2})/4 & 0 & 0 \\ & & \text{Symm.} & & 1/2 & 0 \\ & & & & & 1/2 \end{bmatrix}, \tag{4}$$

with

$$\rho^* = (6\sqrt{2}+2)\pi\left(\frac{a}{L}\right)^2, \tag{5}$$

where ρ^* is the relative density of the lattice material, E_s is the Young's modulus of struts. Symbols a and L are the radius and length of cell struts, respectively. According to Eq. (3) for statically indeterminate lattices, the relationship between macro stresses $\boldsymbol{\sigma}$ and the strut force f_{ij} of each strut is derived as

$$\mathbf{f} = \begin{bmatrix} f_{12} \\ f_{14} \\ f_{15} \\ f_{25} \\ f_{35} \\ f_{45} \\ f_{24} \end{bmatrix} = \frac{\sqrt{2}L^2}{2} \begin{bmatrix} \frac{6+5\sqrt{2}}{8+6\sqrt{2}} & -\frac{2+\sqrt{2}}{8+6\sqrt{2}} & -\frac{1+\sqrt{2}}{4+3\sqrt{2}} & 0 & 0 & 0 \\ -\frac{2+\sqrt{2}}{8+6\sqrt{2}} & \frac{6+5\sqrt{2}}{8+6\sqrt{2}} & -\frac{1+\sqrt{2}}{4+3\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{2+\sqrt{2}}{4+3\sqrt{2}} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{2+\sqrt{2}}{4+3\sqrt{2}} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 0 & \frac{1}{2} & \frac{2+\sqrt{2}}{4+3\sqrt{2}} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{2+\sqrt{2}}{4+3\sqrt{2}} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{3-\sqrt{2}}{4+3\sqrt{2}} & -\frac{3-\sqrt{2}}{4+3\sqrt{2}} & 0 & -\frac{4+2\sqrt{2}}{4+3\sqrt{2}} & 0 & 0 \end{bmatrix} \sigma. \tag{6}$$

The failure of the macro lattice material is mainly caused by the yield of micro struts under tension or compression. While the struts are slender enough, the Euler buckling strength would be smaller than the yield strength. At that time the failure of the macro lattice material comes from the elastic buckling of struts. The micro-failure modes would be suggested by

$$f_{\mathrm{ij}} = \begin{cases} \sigma_{\mathrm{Y}}\pi a^2, & \text{yield by tension} \\ -\sigma_{\mathrm{Y}}\pi a^2, & \text{yield by compression} \\ -\sigma_{\mathrm{E}}\pi a^2, & \text{buckling by compression} \end{cases} \tag{7}$$

where symbols σ_{Y} and σ_{E} denote the yield strength and the elastic buckling strength, respectively. According to Eqs. (6) and (7) one draws the conclusion that initial yield surfaces under general macroscopic loadings are composed of the envelope of at most fourteen super-planes in the stress space as follows:

$$\left\{ \begin{matrix} \frac{16-3\sqrt{2}}{2}\sigma_x + \frac{4-5\sqrt{2}}{2}\sigma_y + (2\sqrt{2}-5)\sigma_z \\ \frac{4-5\sqrt{2}}{2}\sigma_x + \frac{16-3\sqrt{2}}{2}\sigma_y + (2\sqrt{2}-5)\sigma_z \\ \frac{6+\sqrt{2}}{2}\sigma_z + (5\sqrt{2}-4)\sigma_{xy} + (1+3\sqrt{2})\sigma_{yz} + (1+3\sqrt{2})\sigma_{zx} \\ \frac{6+\sqrt{2}}{2}\sigma_z - (5\sqrt{2}-4)\sigma_{xy} + (1+3\sqrt{2})\sigma_{yz} - (1+3\sqrt{2})\sigma_{zx} \\ \frac{6+\sqrt{2}}{2}\sigma_z + (5\sqrt{2}-4)\sigma_{xy} - (1+3\sqrt{2})\sigma_{yz} - (1+3\sqrt{2})\sigma_{zx} \\ \frac{6+\sqrt{2}}{2}\sigma_z - (5\sqrt{2}-4)\sigma_{xy} - (1+3\sqrt{2})\sigma_{yz} + (1+3\sqrt{2})\sigma_{zx} \\ (41-30\sqrt{2})\sigma_x + (41-30\sqrt{2})\sigma_y + (8-10\sqrt{2})\sigma_{xy} \end{matrix} \right\}$$

$$= \begin{cases} \rho^*\sigma_{\mathrm{Y}}, \text{for tension} \\ -\rho^*\sigma_{\mathrm{Y}}, \text{for compression}, \\ -\rho^*\sigma_{\mathrm{E}}, \text{for buckling} \end{cases} \tag{8}$$

where σ_{i} denotes the component of the stress tensor in axes x, y and z, respectively.

The equivalent uniaxial yield strength σ_{is} and buckling strength σ_{ib} of the lattice block material are given by

$$\begin{cases} \sigma_{xs} = \sigma_{ys} \approx 0.17\rho^*\sigma_{\mathrm{Y}} \\ \sigma_{xb} = \sigma_{yb} \approx 0.17\rho^*\sigma_{\mathrm{E}} \\ \sigma_{zs} \approx 0.27\rho^*\sigma_{\mathrm{Y}} \\ \sigma_{zb} \approx 0.27\rho^*\sigma_{\mathrm{E}} \end{cases}. \tag{9}$$

The equivalent shearing yield strength σ_{ijs} and buckling strength σ_{ijb} of the lattice block material are given by

$$\begin{cases} \sigma_{xys} \approx 0.163\rho^*\sigma_{\mathrm{Y}} \\ \sigma_{xyb} \approx 0.163\rho^*\sigma_{\mathrm{E}} \\ \sigma_{yzs} = \sigma_{zxs} \approx 0.191\rho^*\sigma_{\mathrm{Y}} \\ \sigma_{yzb} = \sigma_{zxb} \approx 0.191\rho^*\sigma_{\mathrm{E}} \end{cases}. \tag{10}$$

According to Eqs. (4), (9), and (10) the stiffness and the strength are anisotropic.

To check the validity of the theory, FE simulations were performed to calculate the strength and the stiffness in axes z of the lattice block truss materials made from aluminium casting alloys (LM25). A Young's modulus $E_s = 70$ GPa and a yield strength $\sigma_{\mathrm{Y}} = 170$ MPa [2] were used in the analytical predictions and the FE calculations. The comparison between the analytical and FE predictions of the strength was shown in Fig. 3. Excellent agreement between the FE and analytical calculations was revealed, which solidified the validity of the equivalent continuum method. Thus the method can be used to predict the yield surfaces of the lattice truss materials.

3. Yield surfaces

According to Eq. (8), the yield surface of the lattice block material in macroscopic (σ_x, σ_y, σ_z) space is sketched in Fig. 4. The whole yield surface is made up of eight intersecting yield planes. These planes are associated with tensile and compressive yield modes

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