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Determination of potential function for uncoupled-plane contact problems

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Abstract

In this paper, the elastic contact problem of an asymmetrical shallow wedge with a half-plane is considered. The potential function of the contact problem has been obtained through a simple approach. This approach simplifies the routine method of determination of potential functions in elastic contact problems. Then this approach was utilized to solve the aforementioned geometry under frictionless and frictional condition. Utilization of the new relation not only simplifies the procedure of solving contact problems but also is very helpful for fretting fatigue studies.

Keywords: Contact mechanics; Potential function; Asymmetrical wedge

1. Introduction

Contact problems are very important in solid mechanics due to their occurrences in different physical applications. Also, contact is the principal method of applying loads to the mechanical parts and the resulting stress concentration is often the most critical point in the contacting bodies. Historically, the development of the subject stems from the famous paper of Hertz [1] giving the solution for the frictionless contact of the two elastic cylinders. Since 1882, the subject of contact mechanics has been considered by many researchers and developed too much. Detail solutions in elastic contact problems have been developed by Gladwell [2], Johnson [3] and Hills [4] for different geometries and situations. Muskhelishvili has introduced a complex potential function to solve plane contact problems [5]. In order to achieve the potential function, one needs to integrate a complex integral and the more complicated contact geometry, the more complex mathematical procedure will be needed. Therefore, the potential function of a few contact problems has been obtained directly through the integration. For a contact problem with complicated geometry, there is a routine procedure for the analytical calculation of the potential function as follows:

1. Finding the pressure and shear distribution functions from contact equations.
2. Achieving the Chebyshev expansion of the pressure and shear distribution functions.
3. Utilizing Muskhelishvili's integral relation and integrating the expanded functions in the complex plane.

However, the above-mentioned method does not give the closed-form potential function because the Chebyshev series are not bounded. The convergence of the Legendre series is more rapid than that of the Chebyshev series so it seems suitable to use the Legendre expansion of the traction distributions instead of the Chebyshev expansion [6]. The other method for calculating the potential function is to use the routine procedure but in the third step, in the Muskhelishvili's integral relation, employing the Bertrand–Poincare's lemma to reverse the order of integration for the simplicity of the integration [7,8]. Jager has intro-

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duced a different method for finding the pressure distribution functions [9]. By arranging the flat punches along the contact profile and superposing the pressure distribution functions of the flat punches, for any contact profile, the pressure distribution function can be obtained in terms of a hyper-geometric function. Additionally, finite elements modelling (FEM) has been utilized to solve contact problems [10,11].

In this paper, a simple approach for determination of the potential function in plane contact problems is introduced and utilized. In this approach, potential function is obtained through a simple relation, $2\Phi(z) = p(z) - \mathrm{i}q(z)$, which is proved later. In order to rectify the applicability and accuracy of this approach, a newly solved contact problem is considered; "the tilted shallow wedge problem" that Sackfield et al. have been solved it in [8]. It could be seen that the potential function obtained through this approach is exactly the same as the potential function that Sackfield et al. had obtained using the Bertrand–Poincare's lemma. Additionally, the potential function of the contact problem has been obtained in the case of frictional contact. We have compared our results with the results obtained through the Legendre series expansion method and the results obtained using the finite element method too. Good agreement between the results shows the accuracy and simplicity of this approach.

In the modelling section of this paper, a generalized plane contact problem has been considered. Then, the new approach for determination of the potential function has been introduced in a separate section. Finally, an asymmetrical shallow wedge shape indentation case is solved and a simple form for the pressure distribution function and contact lengths have been obtained.

2. Modelling

In this section, a generalized plane contact problem is considered. Although it might be repetitive, rewriting it is helpful to expose assumptions and equations essential for introducing the approach of determining the potential function. Consider two elastic bodies S_1 and S_2, which are in contact with each other. According to Fig. 1, the body S_1 occupies the upper half-plane and the body S_2 occupies the lower half-plane. Along the boundaries of the bodies, there might be several contact zones. For the considered contact problem, the following usual assumptions have been made:

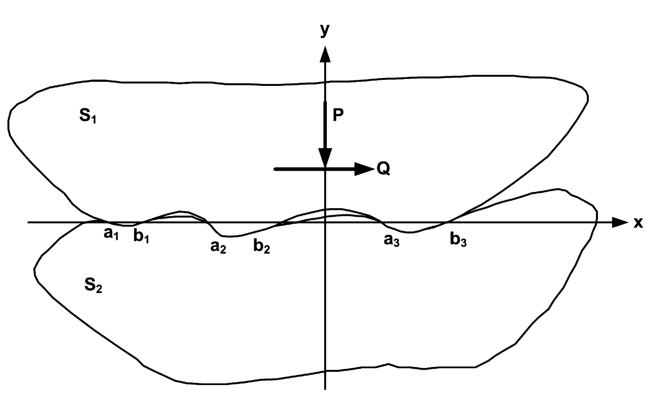

Fig. 1. A generalized plane contact problem.

1. Each body can be approximated with a half-plane to achieve non-conformal contact problem.
2. Each contact length $L_i = [a_i, b_i]$ is too small in comparison with the dimensions of the bodies.
3. There is no rotation of the contacting bodies.
4. The stresses in S_1 and S_2 vanish at infinity.
5. The profiles of the boundaries are known functions before applying the external forces.

The external normal and shear forces produce pressure and shear tractions on the contact surfaces. Along the top surface of the lower body, the pressure distribution function, $p(x)$, and the shear distribution function, $q(x)$, are defined as:

$$p(x) = \sigma_{yy}(x, 0) \tag{1}$$

$$q(x) = \tau_{xy}(x, 0) \tag{2}$$

It is assumed that the above functions $p(x)$ and $q(x)$ vanish at infinity. The vertical overlap function, $h(x)$, and horizontal overlap function, $g(x)$, can be presented as:

$$h(x) = v_1 - v_2 \tag{3}$$

$$g(x) = u_1 - u_2 \tag{4}$$

where v_1 and v_2 are the vertical and u_1 and u_2 are the horizontal components of the displacement of the contacting surfaces of the two bodies. The fundamental contact equations are in the form of the coupled singular integral equations as follows:

$$\frac{1}{A}\frac{\partial h(x)}{\partial x} = \frac{1}{\pi}\int_L \frac{p(\xi)}{(x-\xi)}\,\mathrm{d}\xi - \beta q(x) \tag{5}$$

$$\frac{1}{A}\frac{\partial g(x)}{\partial x} = \frac{1}{\pi}\int_L \frac{q(\xi)}{(x-\xi)}\,\mathrm{d}\xi + \beta p(x) \tag{6}$$

where the composite compliance, A is given by:

$$A = \frac{\kappa_1 + 1}{4\mu_1} + \frac{\kappa_2 + 1}{4\mu_2} \tag{7}$$

$$\beta = \frac{\mu_2(\kappa_1 - 1) - \mu_1(\kappa_2 - 1)}{\mu_2(\kappa_1 + 1) + \mu_1(\kappa_2 + 1)} \tag{8}$$

where $\kappa_i = 3 - 4v_i$ in plane strain condition and $\kappa_i = \frac{3-v_i}{1+v_i}$ in plane stress condition, and L indicates the multiple contact length $L = \sum_{i=1}^{n}[a_i, b_i]$. Also, v_i and μ_i are the Poisson's ratio and the shear modulus of body i ($i = 1, 2$), respectively. If Dundur's constant β is zero, that is to say if the materials of the bodies are similar or the relation $\frac{\mu_2}{\mu_1} = \frac{k_2-1}{k_1-1}$ holds true, the contact equations become decoupled. So, Eqs. (5) and (6) reduce to