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Analytical solution for contact problem between a tilted wedge and half-plane under partial slip condition

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Abstract

Elastic contact problem of a tilted shallow wedge with a half-plane is considered. The problem is solved analytically in either full slip or partial slip conditions. Closed-form expression is given for the Muskhelishvili's potential function of the contact. In the partial slip condition, the locations of the stick and slip zones were given explicitly in terms of the applied external loads and coefficient of friction. The results have been compared with finite elements modelling results.

Keywords: Contact mechanics; Potential function; Tilted wedge; Partial slip

1. Introduction

In design of mechanical components, failure criteria such as avoidance of yield and avoidance of fracture play important role. It is relatively easy to design against some of the failure modes, but design against fracture is more complicated. The reason is that the latter requires not only the determination of the state of stress within an object, but also some speculation on the form, location and origin of initial defects, which often grow by the process of fatigue or other types of failure during the service life of the component and may reach a critical value leading to fracture. The cracks which grow may either originate from some pre-existing macroscopic defect, or, if the component is of high integrity but highly stressed, a region of localized stress concentrations. One of the sources of surface damage which may exist between notionally 'bonded' components is associated with minute relative motion along the interface, brought about usually by tangential loading.

Nucleation of a crack requires a reversing state of shearing stress, in order to provide the environment for dislocation migration, to form a slip band. The more severe the reversing state of stress the more likely a slip band is to form, and therefore reversing contact conditions which are most likely to nucleate a crack; this means that a high coefficient of friction, together with a large tangential force and concentration of contact pressure produce severe condition. Firm evidence of a direct correlation between the incidence of slip and the contest of fretting followed from studies of the contact of elastic spheres. Cattaneo [1], and Mindlin [2], extended the Hertz solution [3] in the theory of elasticity for the contact of two spheres under a normal compressive force to account for the addition of a shearing force less than limiting friction applied parallel to the contact surface. Experimental evidence of the theory was provided by Johnson [4]. Since the Cattaneo work, the subject of contact mechanics have been considered and developed in both mathematical and engineering sights. Detailed solutions in elastic contact problems have been developed by Muskhelishvili [5], Shtaerman [6], Gladwell [7], Johnson [8] and Hills [9] for different geometries and situations.

Several reasons motivate to study contact problem of a wedge shape indenter. For power-law indenters which have

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profiles of the form x^{λ}, solutions are well known for $\lambda = 0, 2$ and higher even values. The solution for $\lambda = 1$, is currently only available in specialized conditions which will be mentioned. Analytical solution of the plane contact problem of a wedge is imperative to understand the stress state induced beneath asperities on rough surfaces, the contacts arising in certain fretting fatigue experiments, the loading imposed by stylus instruments such as surface profile meters [10,11].

The majority of contact solutions encountered in the literature assume symmetrical profiles and symmetrical indentation. This greatly simplifies the solution of the related contact problems. However, non-symmetrical cases may be of considerable interest in many engineering applications. Even with nominally symmetrical contacts, there is a possible effect of a relative rotation, as a result of an applied moment, or of undesired geometrical asymmetry. Non-symmetrical contact problems may occur in situations like hardness testing and surface roughness measurements [12].

Recently, Sackfield et al. have solved the contact problem of a tilted shallow wedge with a half-plane under frictionless condition [13]. This problem is solved also by Adibnazari et al. [14], and it includes a brief description about the full slip condition. Contact problems with frictional force, may lead to either full slip condition or partial slip condition that was formulated by Cattaneo using two similar spheres. Nowadays, the generalized Cattaneo–Mindlin procedure is utilized to solve the partial slip conditions [15–17].

In this paper, the elastic contact problem of a tilted wedge with a half-plane under the presence of a tangential loading such as frictional force is considered. In the modelling section of the paper, the geometry, assumptions and the fundamental equations of the forth mentioning contact problem is introduced. Then, in the solution section, the closed-form of the traction distributions as well as the related potential function is obtained explicitly in both full slip and partial slip conditions. It is also notable that the contact lengths and the location of the stick and slip zones are determined explicitly. The results have been compared with the results obtained by the finite element modeling which good agreement is achieved.

2. Modeling

Consider an asymmetrical wedge shape indenter in contact with a half-plane. According to Fig. 1, the external angles at the apex of the indenter are ϕ_1 and ϕ_2 which should be so small. This assumption allows approximating the wedge as a semi-infinite plane. Due to the tilt angle of the indenter, θ, the external angles at the wedge apex are not equal and ϕ_2 is greater than ϕ_1, so that the contact regions on both sides of the y-axis are not the same and a is greater than b. A constant normal load, P, is pushing the indenter into the half-plane which yields to the indentation amount equal to δ. Add to this, a monotonically increasing tangential load, Q, is applied to the wedge.

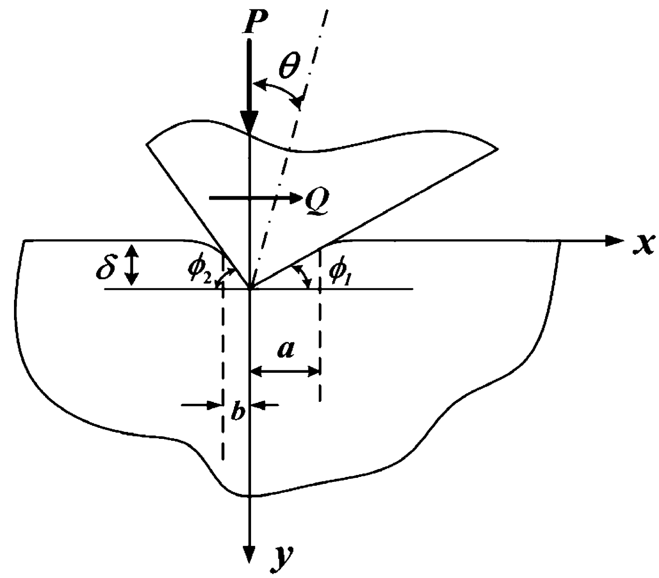

Fig. 1. A tilted wedge in contact with a half-plane.

Along the top surface of the half-plane, the pressure distribution function, $p(x)$, and the shear distribution function, $q(x)$, are defined as

$$p(x) = \sigma_{yy}(x, 0) \tag{1}$$

$$q(x) = \tau_{xy}(x, 0) \tag{2}$$

It is assumed that traction distribution functions vanish at infinity. The vertical overlap function, $h(x)$, and horizontal overlap function, $g(x)$, can be expressed as

$$h(x) = v_1 - v_2 \tag{3}$$

$$g(x) = u_1 - u_2 \tag{4}$$

where v_1 and v_2 are the vertical and u_1 and u_2 are the horizontal components of the displacement field. The fundamental 2D contact equations are in the form of coupled singular integral equations as follow [6]

$$\frac{1}{A}\frac{\partial h(x)}{\partial x} = \frac{1}{\pi}\int_L \frac{p(\xi)}{(x-\xi)}\,\mathrm{d}\xi - \beta q(x) \tag{5}$$

$$\frac{1}{A}\frac{\partial g(x)}{\partial x} = \frac{1}{\pi}\int_L \frac{q(\xi)}{(x-\xi)}\,\mathrm{d}\xi + \beta p(x) \tag{6}$$

where the composite compliance, A, and Dundur's constant, β, are defined as

$$A = \frac{\kappa_1 + 1}{4\mu_1} + \frac{\kappa_2 + 1}{4\mu_2} \tag{7}$$

$$\beta = \frac{\mu_2(\kappa_1 - 1) - \mu_1(\kappa_2 - 1)}{\mu_2(\kappa_1 + 1) + \mu_1(\kappa_2 + 1)} \tag{8}$$

where $\kappa_i = 3 - 4v_i$ in plane strain and $\kappa_i = \frac{3-v_i}{1+v_i}$ in plane stress, and L indicates the contact length $(L = |a| + |b|)$. Also, v_i and μ_i are the Poisson's ratio and the shear modulus of body i ' $(i = 1, 2)$, respectively. If Dundur's constant, β, be zero, that is to say if the materials of the bodies be similar or the relation $\frac{\mu_2}{\mu_1} = \frac{k_2-1}{k_1-1}$ holds true, the contact equa-