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Short communication

A discussion on modeling shape memory alloy embedded in a composite laminate as axial force and elastic foundation

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Abstract

In this paper, the possible error sources of the composite natural frequencies due to modeling the shape memory alloy (SMA) wire as an axial force or an elastic foundation and anisotropy are discussed. The great benefit of modeling the SMA wire as an axial force and an elastic foundation is that the complex constitutive relation of SMA can be avoided. But as the SMA wire and graphite-epoxy are rigidly bonded together, such constraint causes the re-distribution of the stress in the composite. This, together with anisotropy, which also reduces the structural stiffness can cause the relatively large error between the experimental data and theoretical results.

Keywords: Shape memory alloy; Constitutive relation; Constraint; Stress redistribution

1. Introduction: the problem

In Fig. 16 of their paper [1], Epps and Chandra noticed that if the influence of shape memory alloy (SMA) wire is simply modeled as an axial force on the beam, the analysis over-predicts the natural frequency, especially when the temperature is high. The stress inside their SMA wire is tensile, which tends to stiffen the composite structure. Epps and Chandra presented another model which models the SMA wire as an elastic foundation [1]. The elastic foundation property is a function of SMA tension as shown in their appendix [1]. Such elastic foundation modeling dramatically reduce the their computation difference with the experimental data when the temperature is high. While, at low temperature, the two models (axial force and elastic foundation models) hold almost the same relatively large error compared with the experimental data. As it is noticed that the in Epps and Chandra's governing equation (their Eq. (12)) [1], the tensile effect due to SMA wire is only implicitly included in elastic foundation spring constant $k(x)$. Because the SMA wire and the epoxy are bonded, this constraint redistributes the stress inside SMA wire, which affects the actual stress distribution in both SMA wire and epoxy layer. As the SMA wire is modeled as the elastic foundation, Eq. (12) in Epps and Chandra's paper [1] actually is the governing equation for the epoxy layer, in which axial force does not show explicitly. The paper aims to discuss such constraint influence on the stress redistribution, which directly affects the computation of the composite structure natural frequencies. Other factor influencing the computation like the anisotropy is also discussed.

The detailed formulation of 1-D and 2-D models of SMA layer/wire embedded in an orthotropic graphite/epoxy composite matrix layer is presented by Jia and Rogers [2], and Xue and Mei [3]. Their method basically is to apply Hooks Law (constitutive relations) to SMA and composite matrix separately and sum both forces of the SMA layer and composite matrix together to find out the effective Youngs modulus, coefficients of thermal expansion, thus to find out the constitutive relations. The constitutive relations found by this way is widely used in many papers [1,3,4]. The very essence of their traditional method is to assume one strain variable in the constitutive relations, which implicitly includes the constraint. Assuming the continuity of strain in the different layer of the compos-

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ite is the traditional method (and as the result, the stress is discontinuous in the different layer). All the stress redistribution due to the constraint is implicitly included in the assumption. In this paper, the traditional method is re-stated, analyzed and compared with the modeling SMA as an axial force or an elastic foundation. Modeling the SMA layer/wire as an axial force or an elastic foundation can avoid the complex constitutive relation for SMA, which reduces the computation effort. However, the potential danger of neglecting such constraint exists. Traditional method of modeling the SMA layer/wire in the composite is strongly suggested for the analysis of the composite to avoid such danger.

2. Statement and analysis of constitutive relation on the composite material

The following is a simplified 2-D effective constitutive relation of the composite structure presented by Xue and Mei [3]. Fig. 1 shows the schematic diagram of a composite beam/plate with the SMA layer embedded. SMA has the following constitutive relations:

$$\sigma_{s1} = E_s\epsilon_1 + \sigma_r \quad (T > T_{s1},\ \text{SMA activated}), \tag{1}$$

and

$$\sigma_{s1} = E_s(\epsilon_1 - \alpha_{s1}\Delta T) \quad (T < T_{s1},\ \text{SMA inactivated}). \tag{2}$$

For graphite/epoxy composite matrix material, it is

$$\sigma_{m1} = E_{m1}(\epsilon - \alpha_{m1}\Delta T). \tag{3}$$

E_s, E_{m1} are the Young's moduli of SMA and composite matrix material. σ_r is the recovery stress and its detailed expression is given in Liang and Roger's paper [5]. In general, E_s and σ_r are both temperature dependent. The subscript 1 stands for the direction indicated in Fig. 1. T is the temperature and T_{s1} is the temperature when SMA is activated. α_{s1} and α_{m1} are the coefficients of thermal expansion (CTE) for SMA and composite matrix material in 1 direction, respectively. Thermal effects are generally greater at higher temperature [6]. Thus CTE is temperature dependent in general and in the model above, it is taken as a constant. ΔT is the composite temperature difference with the ambient environment. ϵ_1 and ϵ stand for the strain of SMA and the strain of the composite matrix in 1 direction, respectively. Here we deliberately write the strains for the SMA and composite matrix as two variables to emphasize the assumption of one strain variable. In Xue and Mei's paper [3], it is only one strain variable for the strains in SMA and composite matrix layer (Eqs. (1), (2) and (4) in their paper). By assuming such one strain variable, the continuity of strain/displacement of SMA and composite matrix at interfaces is guaranteed and obviously the stress is discontinuous at interfaces. From now on, only one strain variable ϵ_1 is used in 1 direction. It is worth pointing out that ϵ_1 is the total strain [6], which includes mechanical, thermal and recovery ones. The resultant force F_1 in 1 direction is:

$$F_1 = \sigma_1 A_1 = \sigma_{s1}A_s + \sigma_{m1}A_m. \tag{4}$$

A_1 is the total area in 1 direction, A_s is the SMA layer area and A_m is the composite matrix area. Thus, the effective stress for the whole composite beam is:

$$\sigma_1 = \sigma_{s1}V_s + \sigma_{m1}V_m. \tag{5}$$

Here V_s and V_m are called volume fractions and the following expressions hold for them:

$$V_s = \frac{A_s}{A_1}, \quad V_m = \frac{A_m}{A_1}.$$

In the case of $T > T_s$, the constitutive relation (Eq. (5)) can be rewritten as follows:

$$\begin{aligned}\sigma_1 &= (E_s\epsilon_1 + \sigma_r)V_s + E_{m1}(\epsilon_1 - \alpha_{m1}\Delta T)V_m \\ &= E_1\epsilon_1 + \sigma_r V_s - E_{m1}\alpha_{m1}\Delta T V_m.\end{aligned} \tag{6}$$

In the case of $T < T_s$, it is

$$\sigma_1 = E_s(\epsilon_1 - \alpha_s\Delta T)V_s + E_{m1}(\epsilon_1 - \alpha_{m1}\Delta T)V_m. \tag{7}$$

Here E_1 is the effective Young's modulus for the whole composite structure in 1 direction

$$E_1 = E_sV_s + E_{m1}V_m. \tag{8}$$

In 2 direction, there is no recovery stress. The following equations hold for SMA and composite matrix in 2 direction [3]:

$$\sigma_{s2} = E_s(\epsilon_s - \alpha_s\Delta T), \tag{9}$$

and

$$\sigma_{m2} = E_{m2}(\epsilon_m - \alpha_{m2}\Delta T). \tag{10}$$

Here ϵ_s and ϵ_m are the SMA strain and composite matrix strain, respectively. Unlike the case in 1 direction, they are two independent variables in 2 direction. Here SMA is assumed isotropic and composite matrix material is anisotropic. E_{m2} and α_{m2} are the composite matrix Young's modulus and CTE in 2 direction, respectively. Eqs. (9) and (10) can also be rewritten as

$$\epsilon_s = (\sigma_{s2} + \alpha_s E_s\Delta T)/E_s, \tag{11}$$

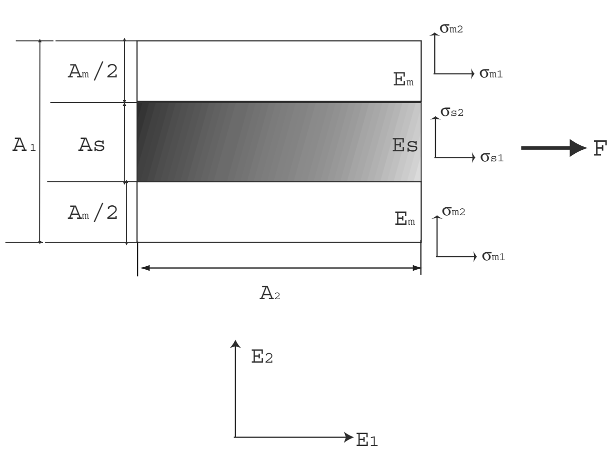

Fig. 1. A schematic diagram of the composite with SMA embedded and its layers' dimensions. The composite is treated as an orthotropic structure.