



# Architecture of attractor determines dynamics on mutualistic complex networks



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## ABSTRACT

A mathematical system of differential equations for the modelization of mutualistic networks in Ecology has been proposed in Bastolla et al. (2007). Basically, it is studied how the complex structure of cooperation interactions between groups of plants and pollinators or seed dispersals affects to the whole network. In this paper we prove existence and characterization of the global attractor associated to the model. The description of the geometrical internal structure of the attractor becomes the proper complex network describing all the possible future scenarios of the phenomena. The arguments show a Morse Decomposition of the attractors, leading to the existence of a global Lyapunov function for the associated gradient semigroup. In particular, we are able to prove topological structural stability of the system, i.e., the associated attracting complex networks are robust under (autonomous and non-autonomous) perturbation of parameters.

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## 1. Introduction

Complex networks driven by mutualistic (or cooperative) relations among nodes are very common in different areas of Science as Ecology, Sociology and Economy. It is probably in Theoretical Ecology where a more abstract formulation of these systems has been reached. In this line, the authors in [1] analyze the net of connections between bipartite graphs representing two kind of species (classified into two sets, plants and animals) and the cooperative links between the differentiated groups (see also [2–5]) (see Fig. 1).

For the analysis of the dynamical properties of the networks, a mathematical model of differential equations has been proposed, which reads as follows: suppose  $P$  is the total number of plants and  $A$  the total number of animals. We suppose that plants (and animals) are in competition and plants and animals have cooperation links. Then, we can write the following system of  $P + A$  differential equations for  $S_{p_i}$  and  $S_{a_i}$

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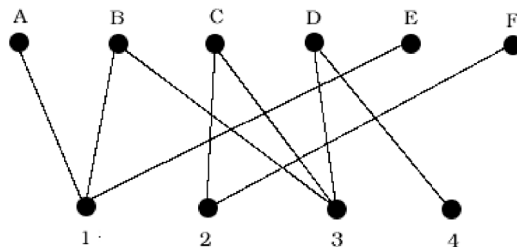


Fig. 1. Typical bipartite graph representing a complex network of mutualistic type. The two sets in the graph represent respectively groups of plants and animals. Each group has competition relations between every two nodes in the same group. The links in the graph represent cooperative relations of nodes between plants and animals of each different group.

the species density populations for the  $i$ th species of plant and of animal respectively:

$$\begin{cases} \frac{dS_{p_i}}{dt} = S_{p_i} \left( \alpha_{p_i} - \sum_{j=1}^P \beta_{p_{ij}} S_{p_j} + \sum_{k=1}^A \gamma_{p_{ik}} S_{a_k} \right) \\ \frac{dS_{a_i}}{dt} = S_{a_i} \left( \alpha_{a_i} - \sum_{j=1}^A \beta_{a_{ij}} S_{a_j} + \sum_{k=1}^P \gamma_{a_{ik}} S_{p_k} \right) \\ S_{p_i}(0) = S_{p_{i0}} \\ S_{a_i}(0) = S_{a_{i0}} \end{cases} \quad (1)$$

for each  $p_i$  for  $1 \leq i \leq P$  and  $a_i$  with  $1 \leq i \leq A$ . Here, the real numbers  $\alpha_{p_i}$  and  $\alpha_{a_i}$  represent the intrinsic growth rates in the absence of competition and cooperation for plants and animals, respectively,  $\beta_{p_{ij}} \geq 0$  and  $\beta_{a_{ij}} \geq 0$  denote the competitive interactions and  $\gamma_{p_{ij}} \geq 0$  and  $\gamma_{a_{ij}} \geq 0$  the mutualistic strengths. For this model, the authors study in [1] (see also [2–6]) how the architecture of a mutualistic network, i.e., the topology of connections between species increases biodiversity in the system. Indeed, it is observed that the more nestedness of the network, the more probability for a richer biodiversity. In particular, and from a dynamical system approach related to (1), this means that *the presence of highly linked cooperative species in the system produces coexistence of species that would go to extinction without them*. But the authors go even further, and explain how the more nestedness species (a topological property of the system), the more capacity of the network to increase biodiversity (a dynamical fact of it).

In this paper we make a full mathematical study of system (1). This will need a careful treatment of parameters in order to avoid blow-up of solutions (see Theorem 2), although, from a dynamical point of view, will not introduce artificial facts into the model. In particular, after a sufficient condition for existence and uniqueness of solutions, which allows us to define a dynamical system  $\{T(t)\}_{t \geq 0}$  for (1), we prove that the system possesses a global attractor,  $\mathcal{A}$ , i.e., a compact invariant set of the phase space determining all the asymptotic behavior of solutions, uniformly on bounded sets (Definition 3). We study the geometrical characterization of this global attractor, which can be described by the union of the unstable manifolds associated to the stationary points for (1). This is a consequence of the two main results of this paper: the dynamical system  $T(t)$  is gradient (Theorem 21), which we prove as a consequence of the system to possess a unique stationary solution which is globally asymptotically stable in the positive cone of solutions (see Theorem 11).

It is important to realize that each equilibrium  $W^*$  is a vector in  $\mathbb{R}^{P+A}$  and that its  $P + A$  components correspond to the  $P + A$  nodes of the phenomenological complex network. In this sense, it is remarkable that each of the stationary points is highlighting a subnet of the former complex network. Indeed, the strictly positive components of each equilibrium point out a subset of nodes and connections of the original network. In particular, the globally stable equilibrium is indeed the complex network of the phenomena showing the future biodiversity of the Ecological system. This is the fact that makes crucial the study of global attractors and its geometrical description for our model. Indeed, for a gradient system, given a finite

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