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Reproduction numbers and the expanding fronts for a diffusion–advection SIS model in heterogeneous time-periodic environment ☆

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HIGHLIGHTS

- A free boundary problem is proposed to describe the expanding of disease in a heterogeneous time-periodic environment.
- The impact of small advection on the spreading of disease is considered.
- The basic reproduction number introduced here depends on the time.
- Numerical simulations are presented to illustrate the effect of small advection, the diffusion rate and the expanding capability.

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ABSTRACT

This paper deals with a simplified SIS model, which describes the transmission of infectious disease in time-periodic heterogeneous environment. To grasp the impact of spatial heterogeneity of environment, temporal periodicity and small advection intensity on the persistence and eradication of the disease, the left and right free boundaries are introduced to represent the expanding fronts. The basic reproduction numbers R_0^D and $R_0^F(\tau)$, which depend on spatial heterogeneity, temporal periodicity, spatial diffusion and advection, are introduced. A spreading–vanishing dichotomy is established and sufficient conditions for the spreading and vanishing of the disease are given. The asymptotic spreading speeds for the left and right fronts are also obtained, and numerical simulations are presented to illustrate the influences of the advection intensity, dispersal rate and expanding capability on the moving fronts.

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1. Introduction

To better understand the transmission of infectious diseases, considerable mathematical models have been formulated and widely investigated. Considering spatial diffusion and environmental heterogeneity, Allen et al. [1] proposed an SIS epidemic reaction–diffusion model subject to a fixed domain

$$\begin{cases} S_t - d_S \Delta S = -\dfrac{\beta(x)SI}{S+I} + \gamma(x)I, & x \in \Omega,\ t > 0, \\ I_t - d_I \Delta I = \dfrac{\beta(x)SI}{S+I} - \gamma(x)I, & x \in \Omega,\ t > 0, \\ \dfrac{\partial S}{\partial \eta} = \dfrac{\partial I}{\partial \eta} = 0, & x \in \partial\Omega,\ t > 0, \end{cases} \tag{1.1}$$

where $S(x,t)$ and $I(x,t)$ stand for the susceptible and infected individuals at location x and time t, respectively, the positive constants d_S and d_I denote the corresponding diffusion rates for the susceptible and infected individuals, $\beta(x)$ and $\gamma(x)$ are positive Hölder continuous functions, which resemble spatial dependent rates of disease contact transmission and disease recovery at x, respectively. The term $\frac{\beta(x)SI}{S+I}$ is the standard incidence of disease.

In some recent works [2–4], Peng and co-workers further investigated the asymptotical behavior and global stability of the endemic equilibrium for system (1.1) subject to the Neumann boundary conditions, and provided intensive and detailed understanding of the impacts of large and small diffusion rates of the susceptible and infected individuals on the persistence and extinction of the disease.

The spreading of the infected fronts is an important factor in describing the transmission of a disease. In recent years, there has been increasing and astonishing interest in understanding the significance that the free boundary plays in the ecological systems. Particularly, Du and Lin [5] proposed a diffusive logistic model in homogeneous environment:

$$\begin{cases} u_t - d u_{xx} = (a - bu)u, & 0 < x < h(t),\ t > 0, \\ u_x(0,t) = u(h(t),t) = 0, & t > 0, \\ h'(t) = -\mu u_x(h(t),t), & t > 0, \\ h(0) = h_0 > 0, \qquad u(x,0) = u_0(x), & 0 \le x \le h_0, \end{cases} \tag{1.2}$$

where the free boundary $x = h(t)$ was used to represent the expanding front of an invasive species. They established the spreading–vanishing dichotomy, and presented the asymptotic spreading speed, which is smaller than the minimal speed of the traveling waves of the corresponding Cauchy problem. Du and Lou [6] considered the free boundary problem with general monostable, bistable and combustion types of $f(u)$, and gave a rather complete description on the long time behavior of the solutions. Later on, many authors extended these results of (1.2) to the general reaction term cases, to different boundary conditions or to higher-dimensional space in a radially symmetric case [7–9]. To reflect the impact of spatial feature, the corresponding free boundary problems were studied in [10–15]. For example, Zhou and Xiao [15] considered the diffusive logistic equation with a free boundary in the heterogeneous environment:

$$\begin{cases} u_t - d u_{xx} = (m(x) - u)u, & 0 < x < h(t),\ t > 0, \\ u_x(0,t) = u(h(t),t) = 0, & t > 0, \\ h'(t) = -\mu u_x(h(t),t), & t > 0, \\ h(0) = h_0 > 0, \qquad u(x,0) = u_0(x), & 0 \le x \le h_0. \end{cases} \tag{1.3}$$

They divided the environment into two different circumstances: strong heterogeneous environment and weak heterogeneous environment, and proved that the spreading–vanishing dichotomy exists only in the weak heterogeneous environment. Biologically, their results show that slow (fast) dispersal is unconditionally (conditionally) favorable for invasive species. Considering the impact of the expanding fronts in the