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Existence of infinitely many solutions of a beam equation with non-monotone nonlinearity

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ABSTRACT

In this paper, we prove a new critical point theorem without the $(PS)^*$ condition or the $(WPS)^*$ condition. Using this theorem, we prove the existence of infinitely many solutions of the following sublinear beam equation

$$\begin{cases} u_{tt} + u_{xxxx} + f(x,t,u) = 0 \quad \text{for } (x,t) \in Q = (0,\pi) \times (0,2\pi) \\ u(0,t) = u(\pi,t) = u_{xx}(0,t) = u_{xx}(\pi,t) \quad \text{for } t \in \mathbb{R} \\ u(x,t) = u(t+2\pi,x) \quad \text{for } t \in (0,\pi) \times \mathbb{R}. \end{cases}$$

Unlike the previous results on this equation, we do not impose any monotone conditions on the variable ξ in the nonlinearity $f(x,t,\xi)$.

1. Introduction and main results

This paper concerns the existence of solutions of the following non-autonomous beam equation:

$$\begin{cases} u_{tt} + u_{xxxx} + f(x,t,u) = 0 \quad \text{for } (x,t) \in Q = (0,\pi) \times (0,2\pi) \\ u(0,t) = u(\pi,t) = u_{xx}(0,t) = u_{xx}(\pi,t) \quad \text{for } t \in \mathbb{R} \\ u(x,t) = u(t+2\pi,x) \quad \text{for } t \in (0,\pi) \times \mathbb{R}. \end{cases} \tag{1.1}$$

Eq. (1.1) has attracted much attention, because it has many important applications in engineering. For example, this equation is used for describing the nonlinear oscillations in suspension bridges (see [1–3]). Critical point theory is a main method for dealing with Eq. (1.1), because it possesses a natural variational structure (e.g., [4–14]). However, when this theory is used for looking for solutions of Eq. (1.1), two difficulties arise. Firstly, since the linear operator $Bu = u_{tt}+u_{xxxx}$ has an infinite-dimensional kernel space (e.g., [8–11]),

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the functional corresponding to Eq. (1.1) does not, in general, satisfy the $(PS)^*$ condition (see Definition 1.3 in this paper). This condition is very important when using the critical point theory for looking for solutions of Eq. (1.1). To overcome this difficulty and to make the functional corresponding to Eq. (1.1) satisfy the $(PS)^*$ condition or the $(WPS)^*$ condition (see Definition 1.4 in this paper), some monotone conditions with respect to the variable ξ are often imposed on the nonlinearity $f(x,t,\xi)$ (e.g., [4,5,8–11,14]). However, to the best of our knowledge, existence results on Eq. (1.1) have not been obtained for the case in which $f(x,t,\xi)$ is non-monotone with respect to the variable ξ. Secondly, as pointed out in [8], it is not known whether there exist L^∞-norm estimates on weak solutions of Eq. (1.1), consequently, it appears invalid to use the truncated function techniques for dealing with Eq. (1.1) when $f(x,t,\xi)$ is sublinear with respect to ξ. This is significantly different from the following wave equation

$$\begin{cases} \Box u = u_{tt} - u_{xx} + g(x,t,u) = 0 \quad \text{for } (x,t) \in Q = (0,\pi)\times(0,2\pi) \\ u(0,t) = u(\pi,t) = 0 \quad \text{for } t \in \mathbb{R} \\ u(x,t) = u(t+2\pi, x) \quad \text{for } t \in (0,\pi)\times\mathbb{R}. \end{cases} \tag{1.2}$$

Although the wave operator $\Box$ also has an infinite-dimensional kernel space, it satisfies the following characterization:

$$\mathrm{Ker}(\Box) = \left\{ p(t+x) - p(t-x) \mid p \in L^2(0,2\pi),\ \int_0^{2\pi} p(s)ds = 0 \right\}.$$

Using this expression, there exist L^∞-norm estimates on weak solutions of Eq. (1.2) (see [15,16]) and consequently, a truncated function technique can be used to obtain nonzero solutions of Eq. (1.2) when $g(x,t,\xi)$ is sublinear with respect to ξ (see [17]).

In this paper, we impose the following conditions on f:

(f_1) $f \in C([0,\pi]\times\mathbb{R}^2,\mathbb{R})$, $f(x,t,-\xi) = -f(x,t,\xi)$, i.e., f is odd in ξ and

$$F(x,t,\xi) > 0 \quad \text{for all } \xi \neq 0 \tag{1.3}$$

where

$$F(x,t,\xi) = \int_0^\xi f(x,t,s)ds.$$

(f_2) There exists $\gamma \in (1,2)$ such that the limit

$$\liminf_{\xi\to 0} \frac{F(x,t,\xi)}{|\xi|^\gamma} > 0 \tag{1.4}$$

holds uniformly for $(x,t) \in Q$.

(f_3) There exist $\beta \in (1,2), D_1 > 0, D_2 > 0$ and $R > 0$ such that for any (x,t,ξ),

$$|f(x,t,\xi)| \le D_1(1 + |\xi|^{\beta-1}), \tag{1.5}$$

and for any (x,t,ξ) with $|\xi| \ge R$,

$$F(x,t,\xi) - \frac{1}{2} f(x,t,\xi)\xi \ge D_2|\xi|^\beta. \tag{1.6}$$