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Existence of positive solutions of four-point BVPs for singular generalized Lane–Emden systems on whole line☆

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ABSTRACT

In this paper, we firstly introduce a model of four-point boundary value problem for generalized singular Lane–Emden systems on whole line. By establishing Green's function $G(t,s)$ for problem $-(\rho(t)x'(t))' = 0, \lim_{t\to-\infty} x(t) - kx(\xi) = 0, \lim_{t\to+\infty} x(t) - lx(\eta) = 0$, we prove that $G(t,s) \geq 0$ under some assumptions which actually generalize a corresponding result in Liu (2004). We then obtain sufficient conditions to guarantee existence of positive solutions of this kind of models. Finally, three examples are given at the end of the paper to illustrate main theorems.

1. Introduction

It is well known that the following systems are called homogeneous and non-homogeneous Lane–Emden systems, respectively,

$$\begin{cases} -\Delta u(x) = v^p(x), \qquad -\Delta v(x) = u^q(x), \quad x \in \Omega, \\ u(x) > 0, \qquad v(x) > 0, \quad x \in \Omega \qquad u(x) = v(x) = 0, \quad x \in \partial\Omega \end{cases} \tag{1.1}$$

and

$$\begin{cases} -\Delta u(x) = v^p(x) + \lambda f(x), \qquad -\Delta v(x) = u^q(x) + \lambda g(x), \quad x \in \Omega, \\ u(x) > 0, \qquad v(x) > 0, \quad x \in \Omega, \qquad u(x) = v(x) = 0, \quad x \in \partial\Omega \end{cases} \tag{1.2}$$

where $p, q \in (1, +\infty)$, Ω is a domain in the n-dimensional Euclidean space $\mathbb{R}^n$, $\Delta = \sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$. Lane–Emden systems (1.1) and (1.2) arise naturally from the study of various nonlinear phenomena, such as pattern

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formation, population evolution, chemical reaction, and so on see [1], and have attracted considerable attention in recent years. In the literature, properties of solutions to (1.1), such as: a priori estimate; existence results; Liouville type theorems; and some symmetric and uniqueness results, were obtained, see [2,3]. Multiplicity results for solutions problem (1.2) have also been proved under certain conditions see [4].

The one-dimensional cases of problems (1.1) and (1.2) are the following forms, respectively:

$$\begin{cases} -u''(t) = v^p(t) + \lambda f(t), \qquad -v''(t) = u^q(t) + \lambda g(t), \quad t \in (-l, l), \\ u(t) > 0, v(t) > 0, t \in (-l, l), \qquad u(-l) = v(-l) = u(l) = v(l) = 0 \end{cases} \tag{1.3}$$

and

$$\begin{cases} -u''(t) = v^p(t), \qquad -v''(t) = u^q(t), \quad t \in (-l, l), \\ u(t) > 0, \qquad v(t) > 0, \quad t \in (-l, l), \qquad u(-l) = v(-l) = u(l) = v(l) = 0. \end{cases} \tag{1.3$'$}$$

The multiplicity of positive solutions of problem (1.3) were studied in [5,3] by making use of the non-degeneracy and uniqueness of solutions of problem (1.3′).

In [6], Liu studied the existence of positive solutions of the following four-point boundary value problem on finite interval

$$y''(t) + a(t)f(y(t)), \quad t \in (0, 1), \qquad y(0) = \mu y(\xi), \qquad y(1) = \beta y(\eta), \tag{1.4}$$

where $0 < \xi \le \eta < 1$, $0 < \mu < \frac{1}{1-\xi}$, $0 < \beta < \frac{1}{\eta}$, and $\mu\xi(1-\beta) + (1-\mu)(1-\beta\eta) > 0$, $a : [0, 1] \to (0, +\infty)$ and $f : [0, +\infty) \to [0, +\infty)$ are continuous functions.

The asymptotic theory of ordinary differential equations is an area in which there is great activity among a large number of investigators. In this theory, it is of great interest to investigate, in particular, the existence of solutions with prescribed asymptotic behavior, which are global in the sense that they are solutions on the whole line. The existence of global solutions with prescribed asymptotic behavior is usually formulated as the existence of solutions of boundary value problems on the whole line.

Motivated by problems (1.3) and (1.4) and paper [6], it is interesting to study existence of positive solutions of four-point boundary value problems for singular differential systems on whole line.

In this paper, we consider the following four-point boundary value problem for second order singular differential system on whole line

$$\begin{cases} [\rho(t)x'(t)]' + f(t, x(t), y(t), x'(t), y'(t)) = 0, \quad a.e., t \in \mathbb{R}, \\ [\varrho(t)y'(t)]' + g(t, x(t), y(t), x'(t), y'(t)) = 0, \quad a.e., t \in \mathbb{R}, \\ \lim\limits_{t \to -\infty} x(t) - ax(\xi) = 0, \qquad \lim\limits_{t \to +\infty} x(t) - bx(\eta) = 0, \\ \lim\limits_{t \to -\infty} y(t) - cy(\xi) = 0, \qquad \lim\limits_{t \to +\infty} y(t) - dy(\eta) = 0, \end{cases} \tag{1.5}$$

where

(a) $-\infty < \xi < \eta < +\infty$, $a, b, c, d \ge 0$ are constants,
(b) f, g defined on $\mathbb{R} \times \mathbb{R}^4$ are nonnegative Carathéodory type functions, and $f(t, 0, u, 0, v) \not\equiv 0$, $g(t, u, 0, v, 0) \not\equiv 0$ on each half line of the forms $(-\infty, t_0]$ or $[t_0, +\infty)$,
(c) ρ, ϱ satisfy that $\rho(t) \in (0, +\infty)$ and $\varrho(t) \in (0, +\infty)$ at almost all $t \in \mathbb{R}$, both $\int_{-\infty}^{+\infty} \frac{du}{\rho(u)}$ and $\int_{-\infty}^{+\infty} \frac{du}{\varrho(u)}$ are finite (convergent), thus differential equations in (1.5) may be singular at some points.

The study of nonlocal boundary value problems for ordinary differential equations (ODEs) was initiated by Il'in and Moiseev [7]. Since then, more general nonlocal boundary value problems (BVPs) were studied by several authors, see the text books [8–10], the papers [11], and the survey papers [12,13] and the references cited there. However, the study on existence of positive solutions of nonlocal boundary value problems for