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Global boundedness in quasilinear attraction–repulsion chemotaxis system with logistic source☆

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ABSTRACT

This paper studies the quasilinear attraction–repulsion chemotaxis system with logistic source $u_t = \nabla\cdot(D(u)\nabla u) - \chi\nabla\cdot(\Phi(u)\nabla v) + \xi\nabla\cdot(\Psi(u)\nabla w) + f(u)$, $\tau v_t = \Delta v + \alpha u - \beta v$, $\tau \in \{0, 1\}$, $0 = \Delta w + \gamma u - \delta w$, in bounded domain $\Omega \subset \mathbb{R}^N$, $N \geq 1$, subject to the homogeneous Neumann boundary conditions, D, Φ, $\Psi \in C^2[0, +\infty)$ nonnegative, with $D(s) \geq (s+1)^p$ for $s \geq 0$, $\Phi(s) \leq \chi s^q$, $\xi s^r \leq \Psi(s) \leq \zeta s^r$ for $s > 1$, and f smooth satisfying $f(s) \leq \mu s(1-s^k)$ for $s > 0$, $f(0) \geq 0$. It is proved that if the attraction is dominated by one of the other three mechanisms with $\max\{r, k, p + \frac{2}{N}\} > q$, then the solutions are globally bounded. Under more interesting balance situations, the behavior of solutions depends on the coefficients involved, i.e., the upper bound coefficient χ for the attraction, the lower bound coefficient ξ for the repulsion, the logistic source coefficient μ, as well as the constants α and γ describing the capacity of the cells u to produce chemoattractant and chemorepellent respectively. Three balance situations (attraction–repulsion balance, attraction–logistic source balance, and attraction–repulsion–logistic source balance) are considered to establish the boundedness of solutions for the parabolic–elliptic–elliptic case (with $\tau = 0$) and the parabolic–parabolic–elliptic case (with $\tau = 1$) respectively.

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1. Introduction

In this paper, we deal with quasilinear attraction–repulsion chemotaxis system

$$\begin{cases} u_t = \nabla\cdot(D(u)\nabla u) - \nabla\cdot\big(\Phi(u)\nabla v\big) + \nabla\cdot\big(\Psi(u)\nabla w\big) + f(u), & (x,t)\in\Omega\times(0,T),\\ \tau v_t = \Delta v + \alpha u - \beta v, & (x,t)\in\Omega\times(0,T),\\ 0 = \Delta w + \gamma u - \delta w, & (x,t)\in\Omega\times(0,T),\\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = \dfrac{\partial w}{\partial n} = 0, & (x,t)\in\partial\Omega\times(0,T),\\ u(x,0)=u_0(x), \qquad v(x,0)=v_0(x), & x\in\Omega, \end{cases} \tag{1.1}$$

where $\Omega\subset\mathbb{R}^N$ $(N\geq 1)$ is a bounded domain with smooth boundary, $\tau\in\{0,1\}$, $\alpha,\beta,\gamma,\delta>0$. The nonlinear nonnegative functions D, Φ, Ψ satisfy

$$D(u),\ \Phi(u),\ \Psi(u)\in C^2[0,\infty), \tag{1.2}$$

$$D(s)\geq (s+1)^p, \quad s\geq 0, \tag{1.3}$$

$$\Phi(s)\leq \chi s^q, \quad s\geq s_0, \tag{1.4}$$

$$\xi s^r \leq \Psi(s)\leq \zeta s^r, \quad s\geq s_0, \tag{1.5}$$

with $\chi,\xi,\zeta>0$, $p,q,r\in\mathbb{R}$, $s_0>1$. The logistic source $f(s)$ smooth on $[0,\infty)$ fulfills

$$f(0)\geq 0 \quad\text{and}\quad f(s)\leq \mu s(1-s^k), \quad s>0 \tag{1.6}$$

with $\mu,k>0$.

In the model (1.1), u, v and w represent the cell density, the concentration of an attractive signal (chemoattractant) and the concentration of a repulsive signal (chemorepellent) respectively. A source of logistic type $f(u)$ is included in (1.1) to prevent an unlimited growth of the cell density. The behavior of solutions would be determined by the interaction among the nonlinear diffusion, attraction, repulsion, and logistic source.

The classical Keller–Segel chemotaxis systems can be obtained by setting $D(u)\equiv 1$, $\Phi(u)=\chi u$ and $\Psi(u), f(u)\equiv 0$ in (1.1), which have been extensively studied since 1970 with rich dynamical properties of solutions established, such as the global existence (boundedness) versus the finite time blow-up of solutions, where the blow-up of solutions results from the attraction mechanism under the mass conservation. Refer to [1–6] and the references therein.

Lately, effects of logistic sources to semilinear chemotaxis systems, that is $D(u)\equiv 1$, $\Phi(u)=\chi u$, $\Psi(u)\equiv 0$ and $k=1$ in (1.1), have been also studied [7–9]. Tello and Winkler [8] proved that if $\mu>\frac{N-2}{N}\chi$, then the solutions of the parabolic–elliptic chemotaxis system are global and bounded, and the same is true for the parabolic–parabolic case, when either $N=2$ [7], or $N\geq 3$ with μ sufficiently large [9]. The general quasilinear chemotaxis systems with logistic source were considered in [10–15]. It has been proved that if $\max\{p+\frac{2}{N},1\}>q$, the solution is global [10,14]; if $q=1$ (the balance case), the global boundedness of solutions is ensured by $\mu>\alpha\chi(1-2/N(1-p)_+)$ for the parabolic–elliptic case (with $\tau=0$) [11], and there exists $\mu_0>0$ such that the solution is globally bounded if $\mu>\mu_0$ for the parabolic–parabolic case (with $\tau=1$) [12,15].

Recently, multi-species and multi-stimulus problems of Keller–Segel systems have been studied more and more as well [16–22]. Among them, Zhang and Li [22] considered currently the homogeneous Neumann problem to the semilinear attraction–repulsion chemotaxis model

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + \xi\nabla\cdot(u\nabla w) + \mu u(1-u), & \text{in} \quad \Omega\times(0,T),\\ 0=\Delta v+\alpha u-\beta v, \qquad 0=\Delta w+\gamma u-\delta w, & \text{in} \quad \Omega\times(0,T) \end{cases} \tag{1.7}$$