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Exponential decay for a locally damped fifth-order equation posed on the line

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ABSTRACT

We prove the exponential decay of the energy related to a locally damped fifth-order equation posed on the whole real line with the initial datum from a bounded set of L^2. A local smoothing effect in H^2 is established, which is essential to obtain the necessary a priory estimates. Moreover, it is shown that arguments used in the article can be applied to prove the exponential decay rate of solutions for the Korteweg–de Vries equation with a similar localized damping term provided the initial data are uniformly bounded in L^2. This last fact improves some previous results.

1. Introduction

The fifth-order equation

$$u_t + u_x + u^{p-1}u_x + u_{xxx} - \gamma u_{xxxxx} = 0, \quad p \geq 2, \tag{1.1}$$

is the nonlinear dispersive PDE that appears in the theory of magneto-acoustic waves in plasma, [1], and in modeling of gravity–capillary water waves, [2]. The refereed equation is also mentioned in [3,4] as a special version of the Benney–Lin equation, [5].

It is well-known that dispersive equations provide soliton-like solutions whose profiles do not change in time. In certain situations, however, it is of interest to control such physical phenomena; the decay of solutions is an essential tool for this purpose. There are various dissipative mechanisms which can be added into the model: second and fourth-order "viscous" terms, nonlocal integral terms, "frictional" damping terms, etc. All these instruments are usually introduced for the KdV or Schrödinger equation; there are few results regarding Eq. (1.1). Moreover, the most part of these studies is mainly concerned with a bounded spatial interval, [6–9].

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The main aim of the present paper is to prove the exponential decay of the energy related to Eq. (1.1) for the case $p = 2, 3, 4$ with so-called "localized" damping term. This kind of dissipation was proposed in [10] to control the KdV equation posed on a bounded interval, and later it was considered for models involved unbounded domains of waves propagation (see, for instance, [11,12] and the references therein).

We are concerned here with the IVP consisting in a locally damped Kawahara equation subject to initial data posed on the whole real line:

$$u_t + u_x + u^{p-1}u_x + u_{xxx} - \gamma u_{xxxxx} + a(x)u = 0, \quad (x,t) \in \mathbb{R} \times \mathbb{R}^+,\ p = 2, 3, \tag{1.2}$$

$$u(x,0) = u_0(x), \quad x \in \mathbb{R}, \tag{1.3}$$

where $u : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$, $\gamma > 0$, and $a(x)$ satisfies the following assumptions:

$$a \in L^\infty(\mathbb{R}) \text{ is a nonnegative function and } a(x) \geq \alpha_0 > 0 \quad \text{for } |x| \geq R,\ R > 0. \tag{1.4}$$

Our main goal is to establish local and global (in time) well-posedness of (1.2), (1.3), the smoothing (hidden regularity) effect, and necessary bounds to prove that $E_0(t) = \frac{1}{2}\|u\|^2_{L^2(\mathbb{R})}(t)$ is exponentially decreasing as $t \to +\infty$. Technically, we mainly follow [11,12], with new issues regarding the unboundedness (in both directions) of the spatial domain, and the higher order of the differential operator.

The paper is organized as follows. In the next section we use the Bourgain approach to prove the local and global well-posedness of (1.2), (1.3) in $L^2(\mathbb{R})$. In Section 3, we establish the exponential decay rate of the energy related to this problem.

2. Notations and well-posedness

2.1. Basic spaces

We use the Sobolev space $H^s(\mathbb{R})$ of order $s \in \mathbb{R}$, defined as

$$H^s(\mathbb{R}) = \{f \in \mathcal{S}'(\mathbb{R}) : (1+\xi^2)^{s/2}\widehat{f}(\xi) \in L^2(\mathbb{R})\}.$$

Here $\mathcal{S}'(\mathbb{R})$ indicates the space of tempered distributions over the Schwartz class $\mathcal{S}(\mathbb{R})$, and $\widehat{f}$ denotes the Fourier transform in $L^2(\mathbb{R})$. We endowed $H^s(\mathbb{R})$ with the norm

$$\|f\|_{H^s} = \left(\int_{\mathbb{R}} (1+\xi^2)^s |\widehat{f}(\xi)|^2 d\xi\right)^{1/2}.$$

If $s = m \in \mathbb{N}$, the space $H^m(\mathbb{R})$ coincides with the usual Sobolev space

$$W^{m,2}(\mathbb{R}) = \{u \in L^2(\mathbb{R}) : \partial_x^j u \in L^2(\mathbb{R}),\ j = 1, 2, \ldots, m.\}$$

with the norm

$$\|u\|_{W^{m,2}} = \left(\sum_{j=0}^{m} \int_{\mathbb{R}} |\partial_x^j u(x)|^2\right)^{\frac{1}{2}}.$$

2.2. The Bourgain space $X^{s,b}$

Consider the following general dispersive equation

$$w_t - iQ(-i\partial_x)w = f(x,t), \tag{2.1}$$

where $Q(-i\partial_x)$ is a linear differential operator in x-variable whose correspondent symbol $Q(\xi)$ is a non-constant real polynomial in one variable, f is a given complex-valued function and $w := w(x,t)$, $(x,t) \in \mathbb{R}\times\mathbb{R}$,