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Persistence in seasonally varying predator–prey systems via the basic reproduction number

Maurizio Garrione[a], Carlota Rebelo[b,*]

[a] Dipartimento di Matematica ed Applicazioni, Università di Milano-Bicocca, Via Cozzi 55, 20125 Milano, Italy

[b] Centro de Matemática, Aplicações Fundamentais e Investigação Operacional, Faculdade de Ciências, Universidade de Lisboa, Campo Grande, Edificio C6, piso 2, 1749-016, Lisboa, Portugal

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We study persistence in general seasonally varying predator–prey models. Using the notion of basic reproduction number R_0 and the theoretical results proved in Rebelo et al. (2012) in the framework of epidemiological models, we show that uniform persistence is obtained as long as $R_0 > 1$. In this way, we extend previous results obtained in the autonomous case for models including competition among predators, prey–mesopredator–superpredator models and Leslie–Gower systems.

1. Introduction

The importance of seasonality in biological systems is unquestionable [1–4]: with almost no exceptions, the dynamics of population communities takes place in periodically varying environments. The main cyclic factors acting in this perspective are both of natural type and human-induced.

Of course, a periodically varying environment can be encountered in several models of different nature, like ecological (e.g., predator–prey) systems and epidemiological ones. Seasonality can be responsible for a wide variety of different complex dynamical features, such as multi-year cycles and subharmonics, quasi-periodic solutions and chaotic behaviors. In particular, it is very natural to wonder, as a first question, if a system exhibiting T-periodicity in the coefficients ($T > 0$) will display periodic dynamics, for instance searching for T-periodic solutions.

* Corresponding author.
E-mail addresses: maurizio.garrione@unimib.it (M. Garrione), mcgoncalves@fc.ul.pt (C. Rebelo).

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Another basic question about biological models involves the long time survival of the species: in particular, we may wonder whether the considered system is uniformly persistent with respect to one (or more) species, namely the number of individuals in that compartment (resp. those compartments) stays positive and bounded away from zero for any positive time. This issue is particularly common in epidemiology, where one usually wants to predict if a disease will go towards extinction or if it will remain endemic. Many authors dealt with this problem giving conditions which guarantee uniform persistence in seasonally forced epidemiological models, see for example [5–7] and the references therein.

In this paper, we will focus on the study of persistence for predator–prey models subject to seasonality (see, e.g., [1,3,8]). As a first remark, notice that it is very frequent – and it will also be one of our starting points – that the dynamics in the prey space exhibits a stable periodic orbit in absence of the predator (for instance, this is a classical fact if the function describing the internal growth of the preys is a logistic one). It is therefore natural to study the linearized system around this orbit, trying to understand the effect of the introduction of a predator. We will employ the usual approach and techniques coming from epidemiology [5,9], in particular we will recall the notion of basic reproduction number R_0 and prove that, under suitable conditions, the predators will go extinct if $R_0 < 1$ and will persist if $R_0 > 1$. As far as we know, this kind of approach is not common in ecology, exceptions are, for example, the papers [10,11], dealing however with autonomous systems.

In autonomous epidemiological models, the basic reproduction number is defined as the expected number of new infectious produced by a single infectious individual, when introduced into a completely susceptible population. For those systems, many classical results state that the threshold value of R_0 in order to determine the qualitative dynamical features of the disease spread is 1: if $R_0 < 1$, then the disease will eventually disappear, while if $R_0 > 1$, it will persist inside the population. In these cases, R_0 can usually be easily computed starting from the coefficients appearing in the equations. On the contrary, it was only in the last years that the concept of basic reproduction number was successfully extended to nonautonomous models [7,9], being defined as the spectral radius of a suitable operator associated with the model. This approach was proved to be quite effective, allowing to obtain several persistence results for nonautonomous systems coming from epidemiology [5–7].

Throughout the present paper, roughly speaking, R_0 may be thought about as the number of predators one predator gives rise during its life, when introduced in a prey population. For periodic predator–prey models, this will be done adapting the definitions given in [7,9]. However, we will be able to proceed a little step further; indeed, not necessarily the population with respect to which persistence is proved corresponds to the predator one, in fact its choice may be not uniquely determined. Nevertheless, thanks to the generality of the setting in [7,9], we will be allowed to disregard whether a compartment is made up by preys or predators, basing our choice only on the dynamics of the system in absence of a certain number of compartments. Indeed, there are examples in which there exists a stable periodic orbit in the space of predators when preys do not exist, and not vice versa (see Section 5). Hence, in this case, preys and predators change role in the definition of R_0. This makes the approach sufficiently flexible and suitable to be applied to a quite large class of models, allowing to find new results for the nonautonomous case, as well as different proofs (often less laborious) of the results already present for autonomous models; in any case, we will always show that $R_0 > 1$ implies uniform persistence, and $R_0 < 1$ leads to extinction. Incidentally, notice that, when $R_0 > 1$, the existence of a periodic motion will follow as a consequence of some results in [12].

The abstract result which will be exploited in our statements, formulated for a general first order system, is given in Section 2; in Sections 3 and 4, we will apply it to infer results about general models of predator–prey type, in presence of one or more predators, respectively. Finally, in Section 5 we will consider a Leslie–Gower model where the role of preys and predators in the definition of R_0 is interchanged. We will constantly explore, after the statements, corresponding models which can be dealt with using our method.