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Homogenization of compressible two-phase two-component flow in porous media

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A B S T R A C T

The paper is devoted to the homogenization of immiscible compressible two-phase two-component flow in heterogeneous porous media. We consider liquid and gas phases, two-component (water and hydrogen) flow in a porous reservoir with periodic microstructure, modeling the hydrogen migration through engineered and geological barriers for a deep repository for radioactive waste. Phase exchange, capillary effects included by the Darcy–Muskat law and Fickian diffusion are taken into account. The hydrogen in the gas phase is supposed compressible and could be dissolved into the water obeying the Henry law. The flow is then described by the conservation of the mass for each component. The microscopic model is written in terms of the phase formulation, i.e. the liquid saturation phase and the gas pressure phase are primary unknowns. This formulation leads to a coupled system consisting of a nonlinear parabolic equation for the gas pressure and a nonlinear degenerate parabolic diffusion–convection equation for the liquid saturation, subject to appropriate boundary and initial conditions. The major difficulties related to this model are in the nonlinear degenerate structure of the equations, as well as in the coupling in the system. Under some realistic assumptions on the data, we obtain a nonlinear homogenized problem with effective coefficients which are computed via a cell problem. We rigorously justify this homogenization process for the problem by using the two-scale convergence.

1. Introduction

The displacement process involving two immiscible fluids is of considerable importance in ground water hydrology and reservoir engineering. Water and soil pollution, CO_2 storage, enhanced oil recovery and nuclear waste management are typical examples of two-phase porous media flows with obvious high societal relevance.

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In this paper, we focus our attention on the modeling of immiscible compressible two-phase flow through heterogeneous reservoirs in the framework of the geological disposal of radioactive waste. The long-term safety of the disposal of nuclear waste is an important issue in all countries with a significant nuclear program. One of the solutions envisaged for managing waste produced by nuclear industry is to dispose the radioactive waste in deep geological formations chosen for their ability to delay and to attenuate possible releases of radionuclides in the biosphere. Repositories for the disposal of high-level and long-lived radioactive waste generally rely on a multi-barrier system to isolate the waste from the biosphere. The multibarrier system typically comprises the natural geological barrier provided by the repository host rock and its surroundings and an engineered barrier system, i.e. engineered materials placed within a repository, including the waste form, waste canisters, buffer materials, backfill and seals, for more details see for instance [1]. An important task of the safety assessment process is the handling of heterogeneities of the geological formation.

In the frame of designing nuclear waste geological repositories, a problem of possible two-phase flow of water and gas, mainly hydrogen, appears, for more details see for instance [1]. Multiple recent studies have established that in such installations important amounts of gases are expected to be produced in particular due to the corrosion of metallic components used in the repository design, see e.g. [2,3] and the references therein. The French Agency for the Management of Radioactive Waste (Andra) [4] is currently investigating the feasibility of deep geological disposal of radioactive waste in an argillaceous formation. A question related to the long-term performance of the repository concerns the impact of the hydrogen gas generated in the wastes on the pressure and saturation fields in the repository and the host rock.

For modeling such flow problems, there are always multiple length scales in the physical coefficients for the governing equations. On the other hand, the size of the repository model prohibits a full fine scale simulation over many time steps, even with the advent of modern computers and parallel computing technology. Therefore, a compromise has to be made between desired accuracy and available computer resources. The standard compromise is to upscale the coefficients which allows the use of a coarse computational grid.

The upscaling or homogenization of multiphase flow through heterogeneous porous media has been a problem of interest for many years and many methods have been developed. There is an extensive literature on this subject. We will not attempt a literature review here, but merely mention a few references. Here we restrict ourself to the mathematical homogenization method as described in [5] for flow and transport in porous media. The periodic homogenization method has the advantage of mathematical rigorousness and allows us to establish explicit connections between processes at various scales, see for instance [5] and the references therein. The method allows the effective parameters to be determined from the knowledge of the geometrical structure of the unit period and its heterogeneities by solving appropriate local problems. The results of this procedure can easily be extended to a non-periodic case by replacing the unit period by a representative elementary volume (REV). The local problem equations are numerically solved and effective parameter values are calculated based on the obtained solutions. To solve these local equations we have to assume a set of boundary conditions. The most general case is to assume linear or periodic boundary conditions, for more details see for instance [6,7] and the references therein.

A recent review of the methods developed for incompressible immiscible two-phase flow in porous media and compressible miscible flow in porous media can be viewed in [8,9]. We refer for instance to [10–16] for more information on the homogenization of incompressible, single phase flow through heterogeneous porous media in the framework of the geological disposal of radioactive waste. However, as reported in [17], the situation is quite different for immiscible compressible two-phase flow in porous media, where, only recently few results have been obtained. In [8], we gave a homogenization result for immiscible compressible two-phase, such as water–gas, flow in porous media using the phase formulation, i.e. where the phase pressures and the phase saturations are primary unknowns. The problem is formulated in terms of a coupled system of diffusion–convection equations in a domain with periodic microstructure with rapidly oscillating porosity function and absolute permeability tensor. We have considered the case of a single rock-type model, i.e. we