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3-dimensional flutter kinematic structural stability

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ABSTRACT

Having recalled the kinematic structural stability (ki.s.s) issue and its solution for divergence-type instability, we address the same problem for flutter-type instability for the minimal required configuration of dimensions—meaning 3 degree of freedom systems. We first get a sufficient non optimal condition. In a second time, the complete issue is tackled by two different ways leading to same results. A first way using calculations on Grassmann and Stiefel manifolds that may be generalized for any dimensional configuration. A second way using the specific dimensional configuration is brought back to calculations on the sphere. Differences with divergence ki.s.s are highlighted and examples illustrate the results.

0. Introduction

This paper deals with the so-called kinematic structural stability (ki.s.s.) for the flutter of non conservative elastic discrete systems. In a previous recent paper (see [1]), the ki.s.s. problematic was formulated in its generality and the solution for the divergence criterion for conservative as well as for non conservative elastic discrete systems has also been given by use of two independent ways. The first one has been proposed for some years by using the formula of Schur's complements, using Lagrange multipliers for introducing the kinematic constraints (see for example [2,3]). The second approach [1] is based upon a variational formulation of the divergence criterion and the explicit elimination of Lagrange's multipliers associated to the additional kinematic constraints. Both approaches lead (fortunately!) to the same results: for conservative elastic systems, the ki.s.s. is universal (as it was for long time known) and can be proved by the use of Rayleigh's quotient and Courant's Minimax results: in fact, adding a kinematic constraint on a conservative

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system cannot destabilize an equilibrium position. We may first remark that, in this last phrase as in the whole paper, possible gyroscopic effects (and especially usual stabilization effects of gyroscopic forces) are not taken into account (see [4]). If not, it is then easy to exhibit a counter-example to the sentence about this ki.s.s. property. We may also mention that Tarnai clearly showed (see [5] for example) that this property fails even for conservative systems if additional kinematic constraints change the considered equilibrium position. Kinematic structural stability refers, by definition, to a given equilibrium position of a mechanical system Σ that must not be changed by adding kinematic constraints: only the eventual change of stability of this equilibrium configuration is investigated. On the contrary, as already mentioned by Thompson [6] in 1982 but never systematically investigated before a set of recent papers ([2,3,1] for example), the non universal divergence-type ki.s.s. is characteristic of the nonconservativity of Σ and the main result reads so:
for non conservative systems, the divergence-type ki.s.s. (or more concisely the divergence ki.s.s.) is only conditional according to the second order work criterion: as long as the symmetric part of the stiffness matrix remains definite positive, the ki.s.s. holds and no additional kinematic constraint may destabilize the system by divergence. As soon as the isotropic cone is not nil, the invert image by the stiffness matrix of a vector chosen on this cone provides a constraint that destabilizes the system by divergence.

In this paper, we focus on the flutter criterion. Because this issue is much more difficult, we only deal meanwhile with 3 dof systems subjected to one additional kinematic constraint and because flutter may occur only for at least 2 dof systems, this configuration of dimensions is the minimal one required to question the ki.s.s issue for the flutter instability. Contrary to the divergence ki.s.s issue, we did not find out a pure algebro-geometric reasoning allowing to solve the problem and differential calculations are definitively necessary. The flutter ki.s.s. is brought back to an optimization problem with vector subspaces as optimization variables. That leads us to use differential geometry tools as Grassmann manifolds even if a parametrization through the sphere of the 3 dimensional euclidean space may be used remaining careful that a same two dimensional vector space has two unit normal vectors. Obviously this parametrization could not be used in higher dimensions whereas the reasoning with Grassmann and Stiefel manifolds is more easily generalizable. For precisions on Grassmann and Stiefel manifolds especially for applications to numerical methods and optimization issues see for example [7,8].

Calculations are done here by both methods and show that the flutter ki.s.s. is neither universal nor conditional but must be handled case by case. There are systems $\Sigma = \Sigma_{free}$ where all the associated constrained systems $\Sigma_{\mathcal{C}}$ are more stable than the initial free system meaning that the critical flutter load value p^*_{fl} for the free system Σ_{free} is lower than the critical flutter load value $p^*_{fl,\mathcal{C}}$ for any constrained system $\Sigma_{\mathcal{C}}$. On the contrary, there are systems Σ where at least one associated constrained system $\Sigma_{\mathcal{C}}$ is less stable than the initial free system meaning that the critical flutter load value p^*_{fl} for the free system Σ is higher than the critical flutter load value $p^*_{fl,\mathcal{C}}$ for the considered constrained system $\Sigma_{\mathcal{C}}$.

The paper is organized as follows. In Section 1, the general ki.s.s. problematic and its solution for divergence type instabilities are first quickly summarized. Then the flutter ki.s.s. issue is formalized leading to an optimization problem for a well-defined function Φ on a Grassmann manifold. In Section 2, some calculations, used subsequently, are done which lead to a significant sufficient condition for preserving flutter ki.s.s. This algebraic condition involves spectrum of both symmetric and skew symmetric parts of the operator. In Section 3, the main calculations are done leading the critical points of Φ. As mentioned above, two ways are used that lead to the same results. The first one is more condensed but uses less known tools of differential geometry. It may be generalized to higher dimensional issues. The second way is more usual by use of differential calculations on the sphere but it leads to more complicated calculations and cannot be generalized for flutter ki.s.s. issue in higher dimension. The general results show again that the flutter ki.s.s. is controlled through a competition between the symmetric and skew symmetric parts of a single operator expressed with respect to the stiffness and the mass matrices of the system. In the fourth and last section, numerical calculations for the paradigmatic 3 dof Ziegler system illustrate the general analytic results.