Nonlinear Analysis: Real World Applications 29 (2016) 58–67

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Blow up criterion for the 3D ghost effect system

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ARTICLE INFO

Article history:
Received 4 August 2015
Received in revised form 26 October 2015
Accepted 26 October 2015
Available online 18 November 2015

Keywords:
3D ghost effect system
Blow up criterion
Strong solutions

ABSTRACT

In this paper, we establish a blow up criterion for strong solutions to the three dimensional ghost effect system.

1. Introduction

In this paper, we consider the Cauchy problem for the following ghost effect system [1–3]:

$$\begin{cases} \nabla(\rho\theta) = 0, \\ \partial_t\rho + \nabla\cdot(\rho u) = 0, \\ \partial_t(\rho u) + \nabla\cdot(\rho u\otimes u) + \nabla p = \nabla\cdot\Gamma + \nabla\cdot\tilde{\Gamma}, \\ \partial_t(c_v\rho\theta) + \nabla\cdot(\gamma c_v\rho\theta u) = -\nabla\cdot q, \end{cases} \tag{1.1}$$

which describes the evolution of the density $\rho(t,x)$, velocity $u(t,x)$, temperature $\theta(t,x)$ and pressure $p(t,x)$ of a γ-law gas at the time $t\in\mathbb{R}^+$ and position $x\in\mathbb{R}^n$. Here Γ, $\tilde{\Gamma}$ and q stand for the viscous stress, thermal stress and heat flux, respectively, which are given as follows:

$$\begin{aligned} \Gamma &\triangleq \mu(\theta)\left(\nabla u + (\nabla u)^T - \frac{2}{D}(\nabla\cdot u)I\right), \\ \tilde{\Gamma} &\triangleq \tau_1(\rho,\theta)\left(\nabla^2\theta - \frac{1}{D}(\Delta\theta)I\right) + \tau_2(\rho,\theta)\left(\nabla\theta\otimes\nabla\theta - \frac{1}{D}|\nabla\theta|^2 I\right) \\ &\quad + \tau_3(\rho,\theta)\left(\nabla\rho\otimes\nabla\theta + \nabla\theta\otimes\nabla\rho - \frac{2}{D}\nabla\rho\cdot\nabla\theta I\right), \\ q &\triangleq -\gamma c_v\kappa(\theta)\nabla\theta, \end{aligned}$$

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http://dx.doi.org/10.1016/j.nonrwa.2015.10.008
1468-1218/

where $\mu(\theta) > 0$ is the coefficient of shear viscosity, $\tau_i(\rho, \theta)$ $(i = 1, 2, 3)$ are the transport coefficients arising from kinetic theory, $\gamma > 1$ is the adiabatic exponent, $c_v \geq \frac{D}{2} \geq \frac{n}{2}$ is the specific heat capacity at constant volume, and $\gamma c_v \kappa(\theta) > 0$ is the coefficient of thermal conductivity.

The system (1.1) describes gas dynamical system flows which are induced by temperature variation. This system is non-classical in the sense that cannot be derived from the compressible Navier–Stokes (CNS) system and is formally derived to describe regimes where the (CNS) system is incomplete. In [1,3–5] the system (1.1) has been derived from kinetic equations by the Hilbert expansion method. Recently, Huang et al. [6] showed that the system (1.1) in the case of $n = 1$ is a diffusive limit of the Boltzmann equation. Levermore et al. [7] show that system (1.1) is a low Mach number limit of a dispersive Navier–Stokes system. It is worth noting that all the results involved are investigating the relations between system (1.1) and Boltzmann equation or dispersive Navier–Stokes system. There are few results about system (1.1) itself.

Notice that the first equation in system (1.1) implies that $\rho\theta$ is independent of x and only a function of t. Hence, $\rho\theta$ is given by the boundary or the far fields in the space rather than the initial condition. Without loss of generality, we can assume $\rho\theta = 1$ and impose the far fields conditions that there exist positive constants $\bar{\rho}$ and $\bar{\theta}$ such that

$$\rho \to \bar{\rho} \quad \text{and} \quad \theta \to \bar{\theta} \quad \text{as} \quad |x| \to \infty.$$

Then the Cauchy problem for system (1.1) can be rewritten as the following system in terms of (θ, u, p):

$$\begin{cases} \partial_t \theta + u \cdot \nabla\theta = \theta \nabla \cdot [\kappa(\theta)\nabla\theta], \\ \partial_t u + u \cdot \nabla u + \theta \nabla p = \theta \nabla \cdot \Gamma + \theta \nabla \cdot \tilde{\Gamma}, \\ \nabla \cdot [u - \kappa(\theta)\nabla\theta] = 0, \\ (\theta, u)|_{t=0} = (\theta_0, u_0), \end{cases} \tag{1.2}$$

where

$$\Gamma = \Gamma_\theta(u) \triangleq \mu(\theta)\left(\nabla u + \nabla u^T - \frac{2}{D}(\nabla \cdot u)I\right),$$

$$\tilde{\Gamma} = \hat{\tau_1}(\theta)\left(\nabla^2\theta - \frac{1}{D}(\Delta\theta)I\right) + \hat{\tau_2}(\theta)\left(\nabla\theta \otimes \nabla\theta - \frac{1}{D}|\nabla\theta|^2 I\right)$$

with

$$\hat{\tau_1}(\theta) = \tau_1\left(\frac{1}{\theta}, \theta\right), \qquad \hat{\tau_2}(\theta) = \tau_2\left(\frac{1}{\theta}, \theta\right) - \frac{2}{\theta^2}\tau_3\left(\frac{1}{\theta}, \theta\right).$$

Under the following conditions:

$$\theta \to \bar{\theta} > 0 \quad \text{and} \quad u \to 0 \quad \text{as} \quad |x| \to \infty, \tag{1.3}$$

$$\theta_0 > 0 \quad \text{and} \quad \nabla \cdot [u_0 - \kappa(\theta_0)\nabla\theta_0] = 0. \tag{1.4}$$

Levermore, Sun and Trivisa [2] proved the local well-posedness for system (1.2) in Sobolev spaces. Very recently, Huang and Tan [8] improved the related result in [2] for 2D and 3D ghost effect system and showed that if the initial temperature is close to some equilibrium states, then there exists an unique global strong solution for 2D ghost effect system.

In this paper, we will consider the short time strong solution obtained in [8] and address a criterion that characterizes the first finite singular time. Before presenting our main results, we recall the well-posedness results obtained in [8] and some notations and definitions.

Notation. *Throughout this paper, $H^s(\mathbb{R}^n)$ and $L^p(\mathbb{R}^n), 1 \leq p \leq \infty$ are standard nonhomogeneous Sobolev space in $\mathbb{R}^n$, if there are no confusions, we write as H^s and L^p [9]. A constant depended on parameters α, β, γ be written as $C(\alpha, \beta, \gamma)$.*