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Well-posedness and global existence for a generalized Degasperis–Procesi equation

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ABSTRACT

We first establish the local existence and uniqueness of strong solutions for the Cauchy problem of a generalized Degasperis–Procesi equation in nonhomogeneous Besov spaces by using the Littlewood–Paley theory. Then, we prove the solution depends continuously on the initial data. Finally, we derive a blow-up criterion and present a global existence result for the equation.

1. Introduction

Recently, Novikov in [1] proposed the following integrable quasi-linear scaler evolution equation of order 2:

$$(1-\epsilon^2\partial_x^2)u_t = \partial_x(2-\epsilon\partial_x)(1+\epsilon\partial_x)u^2, \tag{1.1}$$

where $\epsilon \neq 0$ is a real constant. It was shown in [1] that (1.1) possesses a hierarchy of local higher symmetries and the first non-trivial one is $u_\tau = \partial_x[(1-\epsilon\partial_x u)]^{-1}$.

Eq. (1.1) belongs to the following class [1]:

$$(1-\partial_x^2)u_t = F(u, u_x, u_{xx}, u_{xxx}), \tag{1.2}$$

which has attracted much attention on the possible integrable members of (1.2).

The first well-known integrable member of (1.2) is the Camassa–Holm (CH) equation [2]

$$(1-\partial_x^2)u_t = -(3uu_x - 2u_x u_{xx} - uu_{xxx}).$$

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The CH equation can be regarded as a shallow water wave equation [2,3]. It is completely integrable [2,4], has a bi-Hamiltonian structure [5,6], and admits exact peaked solitons of the form $ce^{-|x-ct|}$, $c>0$, which are orbitally stable [7]. It is worth mentioning that the peaked solitons present the characteristic for the traveling water waves of greatest height and largest amplitude and arise as solutions to the free-boundary problem for incompressible Euler equations over a flat bed, cf. [8–11].

The local well-posedness for the Cauchy problem of the CH equation in Sobolev spaces and Besov spaces was discussed in [12–15]. It was shown that there exist global strong solutions to the CH equation [16,12,13] and finite time blow-up strong solutions to the CH equation [16,12,13,17]. The existence and uniqueness of global weak solutions to the CH equation were proved in [18,19]. The global conservative and dissipative solutions of CH equation were discussed in [20,21].

The second well-known integrable member of (1.2) is the Degasperis–Procesi (DP) equation [22]:

$$(1-\partial_x^2)u_t = -(4uu_x - 3u_xu_{xx} - uu_{xxx}).$$

The DP equation can be regarded as a model for nonlinear shallow water dynamics and its asymptotic accuracy is the same as for the CH shallow water equation [23]. The DP equation is integrable and has a bi-Hamiltonian structure [24]. An inverse scattering approach for computing n-peakon solutions to the DP equation was presented in [25]. Its traveling wave solutions were investigated in [26,27].

The local well-posedness of the Cauchy problem of the DP equation in Sobolev spaces and Besov spaces was established in [28–30]. Similar to the CH equation, the DP equation has also global strong solutions [31–33] and finite time blow-up solutions [34,35,31,36,30,32,37,33]. On the other hand, it has global weak solutions [38,34,37,33].

Although the DP equation is similar to the CH equation in several aspects, these two equations are truly different. One of the novel features of the DP different from the CH equation is that it has not only peakon solutions [24] and periodic peakon solutions [37], but also shock peakons [39] and the periodic shock waves [35].

The third well-known integrable member of (1.2) is the Novikov equation [1]

$$(1-\partial_x^2)u_t = 3uu_xu_{xx} + u^2u_{xxx} - 4u^2u_x.$$

The most difference between the Novikov equation and the CH and DP equations is that the former one has cubic nonlinearity and the latter ones have quadratic nonlinearity.

It was shown that the Novikov equation is integrable, possesses a bi-Hamiltonian structure, and admits exact peakon solutions $u(t,x)=\pm\sqrt{c}e^{|x-ct|},\ c>0$ [40].

The local well-posedness for the Novikov equation in Sobolev spaces and Besov spaces was studied in [41–44,42]. The global existence of strong solutions was established in [41] under some sign conditions and the blow-up phenomena of the strong solutions were shown in [44]. The global weak solutions for the Novikov equation were discussed in [45,46].

To our best knowledge, the Cauchy problem of Eq. (1.1) has not been studied yet. In this paper, we mainly study the Cauchy problem of Eq. (1.1). Since Eq. (1.1) has the similar structure with the Degasperis–Procesi equation, we call it as a generalized Degasperis–Procesi equation.

Letting $v(t,x)=u(\epsilon t,\epsilon x)$, then one can transform Eq. (1.1) into the following equivalent form:

$$u_t - u_{txx} = \partial_x(2-\partial_x)(1+\partial_x)u^2, \quad t>0,\ x\in\mathbb{R}. \tag{1.3}$$

Next, we will investigate the following Cauchy problem:

$$\begin{cases} u_t - 2uu_x = \partial_x(1-\partial_x^2)^{-1}(u^2+(u^2)_x), & t>0,\ x\in\mathbb{R},\\ u(0,x)=u_0(x), & x\in\mathbb{R}. \end{cases} \tag{1.4}$$

In this paper, using the Littlewood–Paley theory, we first establish the local existence and uniqueness of solutions to (1.4) in nonhomogeneous Besov spaces. For the stability of the solution, lots of papers just