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Renormalized solutions in thermo-visco-plasticity for a Norton–Hoff type model. Part I: The truncated case

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A B S T R A C T

We prove existence of global in time strong solutions to the truncated thermo-visco-plasticity with an inelastic constitutive function of Norton–Hoff type. This result is a starting point to obtain renormalized solutions for the considered model without truncations. The method of our proof is based on Yosida approximation of the maximal monotone term and a passage to the limit.

1. Formulation of the problem and main result

Our study is directed to mathematical analysis of thermo-visco-plasticity (for derivation we refer to [1–3]). This means to problems from the theory of inelastic deformations in which the temperature affects the visco-plastic response of the considered material. Let us assume that the elastic constitutive stress–strain relation has the form

$$\sigma = \mathbb{C}(\varepsilon(u) - \varepsilon^p) - f(\theta)\mathbb{1}, \tag{1.1}$$

where σ is the Cauchy stress tensor, $\varepsilon(u) = \frac{1}{2}(\nabla u + \nabla^T u)$ is the linearized strain tensor, u is the displacement vector, ε^p is the inelastic strain tensor, θ is the temperature, f is a given function depending on the considered material and $\mathbb{C}$ is the elasticity tensor which we assume to be symmetric and positive definite on the space of symmetric matrices. Notice that the thermal part of the stress $-f(\theta)\mathbb{1}$ is not linearized in the neighborhood of the reference temperature (compare [4–9]). Our motivation for the form of the elastic constitutive relation (1.1) follows the results of [10,11].

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Moreover, we assume that the density of the internal energy e has the simple form $e = c\,\theta + \langle \mathbb{C}^{-1}T, T\rangle$, where $T = \sigma + f(\theta)\mathbb{1}$ and $\langle , \rangle$ denotes the standard inner product on the appropriate Euclidean space. In this case as a consequence of the first principle of thermodynamics we obtain the following form of the heat equation

$$c\rho\,\theta_t - \varkappa\Delta\theta + f(\theta)\mathrm{div}\,u_t = \langle T, \varepsilon^p_t\rangle, \tag{1.2}$$

where $\rho > 0$ is the mass density, $\varkappa > 0$ is the material's conductivity. It is also assumed that the evolution of the inelastic strain tensor ε^p is given in the form

$$\varepsilon^p_t = G(T), \tag{1.3}$$

where G is a given maximal monotone vector field with $G(0) = 0$ (inelastic constitutive equation with only one internal variable ε^p, see [12]). In the main part of the article we will specify G choosing it in a form of the Norton–Hoff model (similar nonlinear flow rule was considered in [13]).

If we consider Eq. (1.2) with the homogeneous Neumann boundary condition and the homogeneous balance of forces $\mathrm{div}\,\sigma = 0$ also with homogeneous boundary conditions then we can conclude that the considering problem possesses a natural semi-invariant function namely the total energy

$$\mathcal{E}(t) = \int_\Omega c\rho\,\theta\,\mathrm{d}x + \frac{1}{2}\int_\Omega \langle \mathbb{C}^{-1}T, T\rangle\,\mathrm{d}x \le \mathcal{E}(0), \tag{1.4}$$

where $\Omega \subset \mathbb{R}^3$ is a bounded domain describing our considered body with boundary of class C^2. In this simple calculation we have used the dissipation inequality, which yields that $\langle T, \varepsilon^p_t\rangle \ge 0$ in the whole deformation process. From the observation (1.4) we see that the temperature is controlled in the space $L^1(\Omega)$ only (with additional information that $\theta \ge 0$). Moreover, in general the term $\langle T, \varepsilon^p_t\rangle$ belongs to $L^1(\Omega)$ only. From these reasons we are going to prove existence of solution in renormalized sense—see for example [14–16,11,17–20]. In the literature there are only some articles with mathematical study of special thermo-visco-plastic models with various modifications (see for example [4–6,21,22]). See also the articles [23,24], where a poroplasticity models are investigated which have a similar structure to the linear thermo-plasticity.

In this article and in the following work [25], we are going to study the therm-visco-plasticity model of the Norton–Hoff type with a damping term, which we interpret as external forces acting on the material and depending on the deformation velocity. Thus the system of equations, which we study in this article is in the form

$$\begin{aligned}
\mathrm{div}_x\,\sigma &= -F - \mathrm{div}\,\mathbb{C}(\varepsilon(u_t)),\\
\sigma &= \mathbb{C}(\varepsilon(u) - \varepsilon^p) - f(\theta)\mathbb{1},\\
\varepsilon^p_t &= |\,\mathrm{dev}(T)|^{p-1}\,\mathrm{dev}(T),\\
T &= \mathbb{C}(\varepsilon(u) - \varepsilon^p),\\
\theta_t - \Delta\theta + f(\theta)\mathrm{div}\,u_t &= |\,\mathrm{dev}(T)|^{p+1},
\end{aligned} \tag{1.5}$$

where $p > 1$ is a given real number and $\mathrm{dev}\,(T) = T - \frac{1}{3}\,\mathrm{tr}\,(T)\cdot\mathbb{1}$ is a deviatoric part of the symmetric matrix T. The function $F : \Omega \times [0, \mathfrak{T}] \to \mathbb{R}^3$ describes the density of the applied body forces. The main idea in the existence theory of renormalized solutions is to study the so called truncated problem and next to prove that the sequence of obtained solutions converges to a renormalized solution.

In this article we study the truncated model only, hence for fixed $\mathfrak{T} > 0$ and $\epsilon > 0$ we have to find the displacement field $u : \Omega \times [0, \mathfrak{T}] \to \mathbb{R}^3$, the temperature of the material $\theta : \Omega \times [0, \mathfrak{T}] \to \mathbb{R}$ and the visco-plastic strain tensor $\varepsilon^p : \Omega \times [0, \mathfrak{T}] \to \mathcal{S}^3_{\mathrm{dev}}$ ($\mathcal{S}^3_{\mathrm{dev}}$ denotes the set of symmetric 3×3-matrices with vanishing trace) satisfying the following system of equations

$$\mathrm{div}_x\,\sigma = -F - \mathrm{div}\,\mathbb{C}(\varepsilon(u_t)),$$