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Existence analysis of an optimal shape design problem with non coercive state equation

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ABSTRACT

In this paper, a shape optimization problem, modelling a welding process, governed by a second order non coercive PDE is considered. The well posedness of the shape optimal design problem is showed using the degree of Leray–Schauder. Then the existence of an optimal solution is proved.

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1. Introduction

Optimal shape design problems are ubiquitous in science, engineering and industrial applications. Indeed, starting with the foundation of PDE-based optimization [1], shape design has became one of the most frequent application in technologies and it is nowadays one main focus of aerodynamics simulation (see, e.g., [2]). A general shape optimization problem reads as

$$\text{Find } \Omega^* \in \Theta, \quad \text{such that } F(\Omega^*) = \min_{\Omega \in \Theta} F(\Omega) \tag{1}$$

where Θ is a suitable family of admissible domains and F is a suitable cost function defined on Θ.

The mathematical questions associated to (1) which are extensively studied by researchers community: the existence of optimal solution, necessary optimality conditions, qualitative and geometric properties of solutions and the effective computation of the shape optimal. Without any claim to be complete and exhaustive we quote the recent books [3–10], where the reader can find a lot of shape optimization problems together with all the necessary details and references.

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In particular, the case where the cost F is of the form

$$F(\Omega) = J(u_\Omega)$$

with $u|_\Omega$ being the solution of some PDE in Ω. In this case, some authors are interested by spectral optimization [11,12,5] and others by integral functionals with different kind of PDE operator's and different class of admissible domains [13–17,9,18]. In this paper we are concerned by the case of integral functionals with the set of admissible domains Θ is made of domains with uniformly Lipschitz boundaries, and a PDE governed by a non coercive operator appearing in modelling of a heat transfer phenomena [19].

In heat transfer analysis for a welding process, the problem of determining the shape of the phase change front and estimating the temperature field within the solid part of the workpiece is naturally described by a 3-D free boundary model governed by a convection–diffusion equation in evolutionary regime. Thanks to some physical simplifications the obtained model in quasi-stationary regime is a 2-D free boundary problem of Stefan type, which is governed by non coercive PDE [20]. This free boundary problem is formulated in [21], as a shape optimal design problem where the associated cost functional measures the gap between the measured and calculated temperature on a fixed part of the workpiece. Approximation of this problem and convergence results are presented in [22,19]. Our goal in the present work is to prove the existence of an optimal solution to this shape optimization problem which is specific to the welding process. At this stage, we underline that the choice of the class of admissible domains with uniformly Lipschitz boundaries is dictated by physical modelling. In fact, the obtained results concern the existence of the optimal domain still holds for more general family of open domains satisfying the uniform cone property. In other hand, we underline that the state problem in this shape optimization problem is governed by a non coercive equation. This complicates the estimation of the uniform extension of the state problem solution's with respect to domain, which is the basic step in the existence analysis of an optimal solution. The proof of the optimal solution begins by showing the existence and uniqueness of the solution of the non coercive state problem using the Leray–Schauder degree. This allows us to conclude that the shape optimization problem is well posed. Then we prove that the solution of state problem is uniformly bounded with respect to admissible domains using a generalized uniform Poincaré inequality result [23] and some Sobolev inequalities [24]. This present work can be considered as a continuation and extension of our note [21]. This paper is organized as follows: in Section 2, we present the optimal design problem. The existence of at least one solution of the shape optimal problem is proved in the last section.

2. Formulation of the problem

We consider the model problem which describes a heat transfer phenomenon appearing in a welding process. It deals only with the solid part of the plate denoted by Ω (see Fig. 1) and delimited by the boundary $\partial\Omega = \Gamma_m \cup \Gamma_0 \cup \Gamma_1 \cup \Gamma$. This welding free boundary problem consists in finding the interface liquid/solid Γ and the temperature field T solution of the optimal shape design problem proposed in [21],

$$\begin{cases} \text{Find } \Omega^* \in \Theta_{ad} \text{ solution of} \\ J(\Omega^*) = \inf\limits_{\Omega \in \Theta_{ad}} J(\Omega) \\ \text{where} \quad J(\Omega) = \dfrac{1}{2} \displaystyle\int_{\Gamma_m} |T(\Omega) - T_m|^2 \, d\sigma \\ \text{and} \quad T = (\Omega) \text{ is the solution of} \\ (1.1) \quad \begin{cases} \mathcal{V}_0 \cdot \nabla T = \nabla \cdot (\lambda \nabla T) + f & \text{in } \Omega \\ \lambda \dfrac{\partial T}{\partial \nu} = 0 & \text{on } \Gamma_0 \cup \Gamma_m \\ T = T_d \quad \text{on } \Gamma_1, \qquad T = T_f & \text{on } \Gamma. \end{cases} \end{cases} \tag{2}$$

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