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Analysis of a dynamic viscoelastic unilateral contact problem with normal damped response[☆]

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ABSTRACT

In this paper we deal with a viscoelastic unilateral contact problem with normal damped response. The process is assumed to be dynamic and frictionless. Normal damping function is modeled by the Clarke subdifferential of a nonconvex and nonsmooth function. First, the variational formulation of this problem is provided in the form of a nonlinear first order variational–hemivariational inequality for the velocity field. Then, based on the surjectivity results for pseudomonotone and maximal monotone operators, we obtain the unique solvability for a new class of abstract evolutionary variational-hemivariational inequalities. Finally, we apply our abstract results to prove the existence of a unique weak solution to the corresponding contact problem.

1. Introduction

Contact mechanics is a very fascinating field of science. It has many applications in our daily lives. For instance, mechanical linkages, braking systems, combustion engines and metalworking, etc. Due to their complexity, contact phenomena lead to mathematical models expressed in terms of strongly nonlinear evolutionary problems. For this reasons, modeling and mathematical analysis of various contact processes are very important, interesting and challenging.

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The theories of variational inequalities and hemivariational inequalities can be considered as two effective mathematical tools to deal with static, quasistatic and dynamic contact problems for various materials. Important results in these theories can be found in several papers and research monographs, see [1–8] and [9–17], among others.

Variational-hemivariational inequalities represent a special class of inequalities, in which both convex and nonconvex functions are involved. Very recently, some publications have appeared to study their applications in solid contact mechanics. In Han et al. [18] and Migorski et al. [15], the authors considered the existence and uniqueness of solutions for two different types variational-hemivariational inequalities and applied these abstract results to study the static and quasistatic frictional contact problems with unilateral constraints, respectively. In Han et al. [19], the authors studied the adhesive unilateral contact between a viscoelastic body and a deformable foundation. They provided a result on unique solvability for a system consisting of a variational-hemivariational inequality and an ordinary differential equation. Unfortunately, the variational-hemivariational inequalities considered in the above works are elliptic. For this reason, we cannot use directly those results to the dynamic unilateral contact problems with the normal damped response condition.

Motivated by the aforementioned contributions, in this paper, we consider a new dynamic viscoelastic contact model with unilateral constraint, in which the contact boundary condition is modeled with the Clarke subdifferential of a nonconvex function. Our contact model leads to the study of a new first order evolutionary variational inequality with a multivalued term, and to variational-hemivariational inequality for velocity field. For these two, we deliver a result on the unique solvability for a class of abstract evolutionary variational-hemivariational inequalities. The results of this paper find many applications in carrying out the variational analysis of various contact models of mechanics. The systematic studies of problems in contact mechanics by exploiting results on inclusions and hemivariational inequalities can be found in recent monograph [14].

This paper is organized as follows. In Section 2 we recall the basic notation and definitions needed in the paper. Section 3 provides the variational formulation of a dynamic frictionless contact problem for viscoelastic material. In Sections 4 and 5 we establish the solvability and unique solvability results for a class of abstract evolutionary variational inequalities with a multivalued term, and for history-dependent variational-hemivariational inequalities, respectively. In Section 6, we apply our abstract approach to the contact problem and deliver a result on its unique solvability. Finally, Section 7 is devoted to the proofs of some auxiliary results.

2. Preliminaries

In this section, for convenience of the reader, we briefly recall the basic notation and some preliminaries on the theory of nonsmooth analysis and monotone type operators. For more details, see e.g. [20,14,16,21].

Let $(V, \|\cdot\|_V)$ be a reflexive Banach space, V^* denote its dual and let $\langle\cdot,\cdot\rangle_{V^*\times V}$ be the duality pairing between V^* and V.

A single-valued operator $A\colon V\to V^*$ is called pseudomonotone if it is bounded and for all $u_n\in V$, $n=1,2,\ldots$, $u_n\to u$ weakly in V and $\limsup\langle Au_n, u_n-u\rangle_{V^*\times V}\le 0$, we have

$$\langle Au, u-v\rangle_{V^*\times V}\le \liminf\langle Au_n, u_n-v\rangle_{V^*\times V}\quad \text{for all } v\in V.$$

A multivalued operator $B\colon V\to 2^{V^*}$ is called pseudomonotone if

(a) B has nonempty, weakly compact and convex values,

(b) B is upper semicontinuous from every finite-dimensional subspace of V to V^* endowed with weak topology,