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On the inviscid limit of the three dimensional incompressible chemotaxis-Navier–Stokes equations

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A R T I C L E I N F O

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In this paper, we study the inviscid limit of the 3D chemotaxis-Navier–Stokes equations and establish the convergence rate of the inviscid limit for vanishing diffusion.

1. Introduction

We consider the chemotaxis-Navier–Stokes equations in $\mathbb{R}^d$, $d = 2, 3$

$$\begin{cases} \partial_t n + u \cdot \nabla n - \Delta n = -\nabla \cdot (\chi(c) n \nabla c), \\ \partial_t c + u \cdot \nabla c - \Delta c = -f(c) n, \\ \partial_t u + \kappa(u \cdot \nabla u) - \Delta u + \nabla P = -n \nabla \phi, \\ \nabla \cdot u = 0, \\ n(0,x) = n_0(x), \qquad c(0,x) = c_0(x), \qquad u(0,x) = u_0(x). \end{cases} \tag{1.1}$$

Here the unknowns are $n = n(x,t) : \mathbb{R}^d \times \mathbb{R}^+ \to \mathbb{R}^+$, $c = c(x,t) : \mathbb{R}^d \times \mathbb{R}^+ \to \mathbb{R}^+$, $u(x,t) : \mathbb{R}^d \times \mathbb{R}^+ \to \mathbb{R}^d$, and $P = P(x,t) : \mathbb{R}^d \times \mathbb{R}^+ \to \mathbb{R}$, denoting the cell density, chemical concentration, velocity field and pressure of the fluid, respectively. The constant κ is related to the strength of nonlinear fluid convection. The given potential function $\phi = \phi(x)$ represents the gravitational potential. Different functional forms of χ and f are meaningful, according to various conceivable threshold effects and saturation mechanism. In general, χ, f and ϕ are supposed to be sufficiently smooth given function.

The system (1.1) describes a biological process, in which bacteria move towards higher concentration of oxygen which they consume. Meanwhile a gravitational effect on the motion of the fluid is produced by the

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heavier bacteria, and a convective transport of both cells and oxygen is happened through the water. One can refer to [1,2] for more details.

Due to the significance of the biological background, many mathematicians have studied this model and made more progress in the past years. Briefly, the local existence of weak solutions for problem (1.1) is obtained by A. Lorz [1] in a bounded domain in $\mathbb{R}^d$, $d = 2, 3$. Moreover, M. Chae, K. Kang and J. Lee [3] prove the local well-posedness and blow-up criterion of smooth solutions of (1.1) in the framework H^m with $m \geq 3$ in $\mathbb{R}^d$, $d = 2$, 3. Then the result is extended by Q. Zhang to Besov spaces [4]. As $\kappa = 0$, a global existence result of weak solutions are obtained in [5] for (1.1) with smallness assumptions on either $\nabla\phi$ or the initial data c. The key ingredient of their proof is to establish *a priori* estimates involving energy type functionals. Subsequently, J.G. Liu and A. Lorz [6] remove the above smallness conditions and prove the global existence of weak solutions to the 2D Navier–Stokes version of (1.1) with $\kappa = 1$ for arbitrarily large initial data, under the basically the same assumptions on χ and f made in [5]. In the recent paper [7], M. Winkler proves that the system (1.1) admits a unique global classical solutions in a bounded convex domain with smooth boundary in $\mathbb{R}^2$ under the weaker assumptions on χ, f, ϕ than [5,6]. Then the authors [8] establish some new estimates and prove the global well-posedness for 2D chemotaxis-Navier–Stokes equations in $\mathbb{R}^2$.

For the three-dimensional chemotaxis-Navier–Stokes equations, the global classical solutions near constant steady states are constructed in [5] for the system (1.1) with $\kappa = 1$. When $\kappa = 0$, M. Winkler [7] shows that (1.1) possesses at least one global weak solution. However, whether solutions of (1.1) with large initial data exist globally or may blow up appears to remain an open problem.

Besides the well-posedness theory for solutions to the incompressible chemotaxis-Navier–Stokes system (1.1), there are some works on a qualitative behavior of such solution. Tuval et al. [2] report the emergence of patterns on intermediate time scales by numerical analyzing. Moreover, some numerical studies concerning a behavior of such solution can be found in [9,10]. Recently, for a bounded convex domain Ω, M. Winkler [11] asserts that a solution of the two-dimensional chemotaxis-Navier–Stokes system stabilizes to the spatially uniform equilibrium $(\overline{n}_0, 0, 0)$ with $\overline{n}_0 = \frac{1}{|\Omega|}\int_\Omega n_0(x)\,dx$ in the sense of $L^\infty(\Omega)$.

Recently, several authors of chemotaxis literature have considered the chemotaxis-Navier–Stokes system with nonlinear diffusion governed by

$$\begin{cases} \partial_t n + u\cdot\nabla n - \Delta n^m = -\nabla\cdot(\chi(c)n\nabla c), & (t,x)\in\mathbb{R}^+\times\mathbb{R}^d,\ d=2,3,\\ \partial_t c + u\cdot\nabla c - \Delta c = -f(c)n, \\ \partial_t u + \kappa(u\cdot\nabla u) - \Delta u + \nabla P = -n\nabla\phi, \\ \nabla\cdot u = 0, \\ n(0,x) = n_0(x), \qquad c(0,x) = c_0(x), \qquad u(0,x) = u_0(x). \end{cases} \tag{1.2}$$

This model can be viewed as a closely related variant of (1.1) when Δn is replaced by the porous medium-type expression Δn^m with $m > 1$. From a mathematical point of view, choosing m large should enhance the balancing effect of the nonlinear diffusion term in the first equation in (1.2), so that solutions are likely to remain bounded and global existence. To our knowledge, the paper [12] is first to show the global existence of weak solutions for this problem (1.2) with $\kappa = 0$ in bounded domains $\Omega \subset \mathbb{R}^2$ when $m \in (\frac{3}{2}, 2]$. Moreover, Y. Tao and M. Winkler [13] extend the above result, and prove that global weak solutions exist whenever $m \in (1, \infty)$ and that all solutions remain bounded if the initial data are sufficiently regular. Also, it has been shown in [6] that the global existence of weak solutions are established for the problem (1.1) with $\kappa = 0$ and the prize value $m = \frac{4}{3}$ in $\mathbb{R}^3$ under some additional assumptions on χ and f. This complements a corresponding result in [12] which asserts global weak solvability of (1.1) for any $m \in [\frac{7+\sqrt{217}}{12}, 2]$ and bounded domains $\Omega \subset \mathbb{R}^3$. More recently, Y. Tao and M. Winkler [14] improve this result and prove the global existence of weak solution of (1.2) in $\mathbb{R}^3$ as $m > \frac{8}{7}$ and $\kappa = 0$, but whether the lower bound $\frac{8}{7}$ for m is optimal is not clear.