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Global solutions to nonisentropic hydrodynamic models for two-carrier plasmas

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ABSTRACT

This paper is concerned with nonisentropic hydrodynamic models for two-carrier plasmas, which take the form of Euler equations for conservation laws of mass density, current density and energy density for two-carrier plasmas, coupled to Poisson's equation for self-consistent electronic field. Due to the nonlinear coupling and cancellation between electrons and ions, the expected dissipation rates of densities for two carriers are no longer available in comparison with the one-carrier case, which leads to the lack of exponential stability near constant equilibrium in the whole space. In order to capture the weaker dissipation and obtain global solutions in spatially critical Besov spaces, calculus techniques which have been recently developed in Chemin–Lerner spaces, will be further applied.

1. Introduction

Denote by $n_e = n_e(t,x), u_e = (u_{e1}, u_{e2}, \ldots, u_{eN}), W_e = W_e(t,x)$ (n_i, u_i, W_i, resp.) the mass density, current density and energy density of electrons (ions, resp.), and by $\Phi = \Phi(t,x)$ the electrostatic potential. By applying the moment method and appropriate closure conditions, a hydrodynamic model for plasmas which takes the form of Euler equations for conservation laws of n_a, u_a and $W_a (a = e, i)$ coupled to Poisson's equation of Φ, can be derived from the semiclassical Boltzmann–Poisson equation, see [1] for more explanation. Precisely,

$$\begin{cases} \partial_t n_a + \operatorname{div}(n_a u_a) = 0, \\ \partial_t (n_a u_a) + \operatorname{div}(n_a u_a \otimes u_a) + \nabla p_a = -q_a n_a \nabla \Phi - n_a u_a, \\ \partial_t W_a + \operatorname{div}(u_a W_a + u_a p_a) = -q_a n_a u_a \cdot \nabla \Phi - (W_a - W^*), \\ \Delta \Phi = n_e - n_i, \end{cases} \tag{1.1}$$

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for $(t,x) \in [0,+\infty) \times \Omega$ ($\Omega = \mathbb{R}^N$ or $\mathbb{T}^N, N \geq 2$), where $q_e = -1$ and $q_i = +1$. To close (1.1), the pressure p_a satisfies the state equation $p_a = n_a T_a$. Denote by W_a the energy density as $W = \frac{n_a|u_a|^2}{2} + \frac{3P_a}{2}$, and $W^* = \frac{3n_a T^*}{2}$ is ambient device energy, where T_e (T_i, resp.) is the temperature of electrons (ions, resp.). $T^* > 0$ is a given ambient temperature, which is assumed to be a constant for simplicity. The symbols ∇, Δ and $\otimes$ are the gradient operator, Laplace operator and the tensor products of two vectors, respectively.

System (1.1) is supplemented by initial conditions for $n_a, u_a, T_a (a = e, i)$ and by the boundary condition for Φ in case that $\Omega = \mathbb{R}^N$:

$$n_a(x,0) = n_{a0}(x), \qquad u_a(x,0) = u_{a0}(x), \qquad T_a(x,0) = T_{a0}(x), \tag{1.2}$$

$$\lim_{|x| \to +\infty} \Phi(t,x) = 0, \quad \text{a.e. } t > 0, \tag{1.3}$$

where the homogeneous boundary condition for Φ means that the plasma is in equilibrium at infinity.

It is not difficult to see that (1.1) becomes an one-carrier nonisentropic hydrodynamic model, if the transport of one carrier (e.g., electrons) is considered inside devices only. As we all know (see, e.g., [1]), (1.1) can simulate physical phenomena effectively, such as velocity overshoot and ballistic effects in the numerical simulation, especially for submicron devices. On the other hand, it reduces the computational price in comparison with Boltzmann–Poisson equations when using numerical simulation. Therefore, (1.1) has recently received increasing attention in statistical physics and applied mathematics, which represents a reasonable compromise between the physical accuracy and the reduction of computational cost.

From the mathematical point of view, (1.1) is a strongly coupled hyperbolic–elliptic system. Furthermore, with the aid of Green's formulation in Remark 1.1, it reduces to a quasilinear hyperbolic system with nonlocal terms. Besides, another partial source terms related to the momentum and energy appear as a hindrance. To the best of our knowledge, without these source terms, the main feature of (1.1) is the finite-time blowup of classical solutions even when initial data are smooth and small. Consequently, it is interesting to explore the large-time behavior of classical solutions by the damping action of the source. In the case of one-carrier, Chen, Jerome and Zhang [2] first investigated the one-dimensional IBV problem and established the global existence and asymptotic decay of classical smooth solutions, which indicated that the relaxation term could prevent the development of shock waves for smooth initial data with small oscillation. For smooth initial data with large amplitude, Wang and Chen [3] showed the weak dissipation role of heat diffusion and relaxation damping, which is not strong enough to prevent the formation of singularities and the development of shock waves. Amster, Varela, Jüngel and Mariani [4] proved the existence of a classical subsonic solution with positive particle density and positive temperature for steady nonisentropic models. Hsiao–Jiang–Zhang [5] studied the Cauchy–Neumann problem in dimension three and established the global exponential stability of small smooth solutions near the constant equilibrium. Subsequently, Li [6] generalized their results to the perturbation problem of the nonconstant equilibrium and got similar results. In [7], Alì discussed the Cauchy problem for the extended thermodynamic model ($N \geq 2$) and achieved the exponential stability of global classical solutions. By using harmonic analysis tools, especially for the Littlewood–Paley decomposition and Bony's para-product formula, the first author [8] particularly focused on the regularity and established the exponential decay of classical solutions (close to equilibrium) in spatially critical Besov spaces independently. Note that some physical parameters, like the momentum, energy relaxation times and Debye length, are normalized to be one in (1.1). If considered, there are rigorous justifications on the combined singular parameter limits, which are widely adopted in the nonequilibrium physics. The interested reader is referred to [7,9–15] and therein references.

For the two-carrier model, it is observed that the dissipative mechanism appear relatively *weaker* in comparison with the one-carrier case. More precisely, the dissipation rates of densities for two carriers are no longer available, which leads to the lack of exponential stability near constant equilibrium in the whole space. Therefore, the low-frequency and high-frequency methods employed in [8] cannot be applied directly