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Existence and asymptotic behavior of positive solutions for a one-prey and two-competing-predators system with diffusion[☆]

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ABSTRACT

A diffusive one-prey and two-competing-predators system under homogeneous Dirichlet boundary conditions is studied. First, we obtain sufficient conditions for the extinction and existence of global attractor of the time-dependent system by means of the comparison principle. Second, we discuss the existence and nonexistence of coexistence states, and give sufficient conditions for the existence of coexistence states by using the fixed point index theory. In addition, we investigate the bifurcation from a double eigenvalue by virtue of space decomposition and implicit function theorem. Finally, some numerical simulations are made to verify and complement the theoretical analysis.

1. Introduction

In recent years, reaction–diffusion equations modeling of various systems have attracted considerable attention in mathematical biology, especially the predator–prey systems with various functional responses and different boundary conditions. In population dynamics, the relationship between predator and their prey plays an important role. From the biological significance of reality, a key issue for a predator–prey system study is whether the various species can coexist. Therefore, the elliptic steady state of predator–prey system becomes the main object being studied.

On the other hand, taking into account spatially inhomogeneous distribution at any given time, we are concerned in this paper with the following three species predator–prey system under homogeneous Dirichlet

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boundary conditions:

$$\begin{cases} u_t - \Delta u = u\left(a - u - \dfrac{\alpha v}{1+k_1\alpha u} - \dfrac{\beta w}{1+k_2\beta u}\right), & x \in \Omega, \quad t > 0, \\ v_t - \Delta v = v\left(\dfrac{p_1\alpha u}{1+k_1\alpha u} - \dfrac{p_1\alpha v}{1+k_1\alpha u} - c_1 w - d_1\right), & x \in \Omega, \quad t > 0, \\ w_t - \Delta w = w\left(\dfrac{p_2\beta u}{1+k_2\beta u} - \dfrac{p_2\beta w}{1+k_2\beta u} - c_2 v - d_2\right), & x \in \Omega, \quad t > 0, \\ u(x,t) = v(x,t) = w(x,t) = 0, & x \in \partial\Omega, \quad t > 0, \\ u(x,0) = u_0(x) \geq 0, \not\equiv 0, \qquad v(x,0) = v_0(x) \geq 0, \not\equiv 0, & x \in \Omega, \\ w(x,0) = w_0(x) \geq 0, \not\equiv 0, & x \in \Omega, \end{cases} \tag{I}$$

where Ω is a bounded domain in R^N with smooth boundary $\partial\Omega$. u, v and w represent the densities of the prey and the two predators respectively. a is the intrinsic growth rate of prey u. α and β measure efficiency of the searching and the capture of predators v, w respectively. k_1 and k_2 represent the handling and digestion rates of predators. p_1 and p_2 represent the efficiency of converting consumed prey into predator births. c_1 and c_2 measure the interspecific competition factors that are interference competition of the predator v on predator w and vice versa. d_1 and d_2 are the death rates of the two predators. $u_0(x), v_0(x)$ and $w_0(x)$ are continuous functions. The parameters $a, \alpha, \beta, k_i, p_i, c_i, d_i$ $(i = 1, 2)$ are positive constants.

The dynamical interactions of a one-prey and two-competing-predators model is proposed as (I). The ODE model corresponding to (I) was presented and studied in [1], where the ecological background of the model was explained in detail and the persistence of the ODE model with $a = 1$ was studied. In (I), the Holling type-II functional response $\frac{u}{1+hu}$ is used to describe feeding of the two predators v and w on prey u. This functional response was proposed by Michaelis–Menten and Holling in studying enzymatic reactions and predator–prey models. Mathematical and mechanistical simplicity is the biggest advantage of the Holling type-II functional response. For more information about the background and applications of the Holling type-II functional response, one can refer to [2–4].

At present, the two species predator–prey models have been studied extensively, see [5–21] and the references therein. However, the dynamics ones have not be well investigated since they are more complicated than those of two species cases. About three species reaction–diffusion systems, one can see [22–33]. Some of these references gave the conditions of existence and nonexistence of positive solutions (see [22–27]). In [22–26,28], the three species predator–prey systems with homogeneous Neumann boundary conditions were discussed. Especially in [25], the existence of positive steady state solution for the three species predator–prey model was discussed by using the bifurcation theory. In [27,29,30], the interaction of species involving persistence and extinction was investigated. For the dynamics of more species interacting models, one can refer to [34–36] and the references therein. The food chain models were studied in [24,30,34–36], for example, the stability of the constant positive steady state solution in [24], and the existence and nonexistence of nonconstant positive steady state solutions for a food chain model are also investigated by means of the fixed point index theory. We point out that little work has been done about system (I) at the moment.

For the sake of simplicity, we denote $k_1\alpha = h_1, k_2\beta = h_2, p_1\alpha = e_1, p_2\beta = e_2$. Then problem (I) is reduced to

$$\begin{cases} u_t - \Delta u = u\left(a - u - \dfrac{\alpha v}{1+h_1 u} - \dfrac{\beta w}{1+h_2 u}\right), & x \in \Omega, \quad t > 0, \\ v_t - \Delta v = v\left(\dfrac{e_1 u}{1+h_1 u} - \dfrac{e_1 v}{1+h_1 u} - c_1 w - d_1\right), & x \in \Omega, \quad t > 0, \\ w_t - \Delta w = w\left(\dfrac{e_2 u}{1+h_2 u} - \dfrac{e_2 w}{1+h_2 u} - c_2 v - d_2\right), & x \in \Omega, \quad t > 0, \end{cases} \tag{1.1}$$