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Existence solutions for second order Hamiltonian systems

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ABSTRACT

We study the existence of periodic solutions for a second order non-autonomous dynamical system containing variable kinetic energy terms. Subquadratic problems and superquadratic problems are both considered.

1. Introduction and main results

We consider the following problem. One wishes to solve

$$-\ddot{x}(t) = B(t)x(t) + \nabla_x V(t, x(t)), \tag{$\mathcal{H}$}$$

where

$$x(t) = (x_1(t), \dots, x_n(t))$$

is a map from $I = [0, T]$ to $\mathbb{R}^n$ such that each component $x_j(t)$ is a periodic function in H^1 with period T, and the function $V(t, x) = V(t, x_1, \dots, x_n)$ is continuous from $\mathbb{R}^{n+1}$ to $\mathbb{R}$ with

$$\nabla_x V(t, x) = (\partial V/\partial x_1, \dots, \partial V/\partial x_n) \in C(\mathbb{R}^{n+1}, \mathbb{R}^n). \tag{1}$$

For each $x \in \mathbb{R}^n$, the function $V(t, x)$ is periodic in t with period T. The elements of the symmetric matrix $B(t)$ are to be real-valued functions $b_{jk}(t) = b_{kj}(t)$, and each function is to be periodic with period T. We will consider each function to be defined on the interval I.

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The periodic non-autonomous problem

$$-\ddot{x}(t) = \nabla_x V(t, x(t)) \tag{2}$$

has an extensive history in the case of singular systems (cf., e.g., Ambrosetti–Coti Zelati [1]). The first to consider it for potentials satisfying (1) were Berger and Schechter [2] in 1977. They proved the existence of solutions to (2) under the condition that

$$V(t,x) \to \infty \quad \text{as } |x| \to \infty$$

uniformly for a.e. $t \in I$. Subsequently, Mawhin–Willem [3], Tang [4–8] Tang–Wu [9–11] and others proved existence under various conditions (cf. the references given in these publications). Most previous work considered the case when $B(t) = 0$. Ding and Girardi [12] considered the case of $(\mathcal{H})$ when the potential oscillates in magnitude and sign, and found conditions for solutions when the matrix $B(t)$ is symmetric and negative definite and the function $V(x)$ grows superquadratically and satisfies a homogeneity condition. Antonacci [13,14] gave conditions for existence of solutions with stronger constraints on the potential but without the homogeneity condition, and without the negative definite condition on the matrix. Generalizations of the above results are given by [13,15–26].

In this paper, we shall study this problem under the following assumptions. Our assumption on $B(t)$ is:

(B) Each component of $B(t)$ is an integrable function on I, i.e., for each j and k, $b_{jk}(t) \in L^1(I)$.

Although this assumption is very weak, it is sufficient to allow us to find an extension $\mathcal{D}$ of the operator

$$\mathcal{D}_0 x := -\ddot{x}(t) - B(t)x(t)$$

having a discrete, countable spectrum consisting of isolated eigenvalues of finite multiplicity with a finite lower bound $-L$

$$-\infty < -L < \lambda_1 < \lambda_2 < \cdots < \lambda_l < \cdots.$$

($\mathcal{D}$ is defined in the next section.) Let λ_l be the first positive eigenvalue of $\mathcal{D}$. We allow $\lambda_{l-1} = 0$. For the superquadratic case, we assume:

(V1) Assume

$$2V(t,x) \geq \lambda_{l-1}|x|^2, \quad t \in I, x \in \mathbb{R}^n$$

and there are positive constants $\mu < \lambda_l$ and δ such that

$$2V(t,x) \leq \mu |x|^2, \quad |x| \leq \delta, x \in \mathbb{R}^n.$$

(V2) There exists $q > 2$ such that

$$\lim_{|x|\to\infty} \frac{V(t,x)}{|x|^q} < +\infty$$

uniformly for all $t \in I$.

(V3) $\lim_{|x|\to\infty} \frac{V(t,x)}{|x|^2} = +\infty$ uniformly for all $t \in I$.

(V4) There exists $\theta \geq 1$ such that

$$\theta F(t,x) \geq F(t,sx) \quad \forall (t,x) \in I \times \mathbb{R}^n,\ \forall s \in [0,1],$$

where $F(t,x) = (x, \nabla_x V(t,x)) - 2V(t,x)$.