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Periodic solutions to a class of biological diffusion models with hysteresis effect

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ABSTRACT

This paper is concerned with a class of biological models which consists of a nonlinear diffusion equation and a hysteresis operator describing the relationship between some variables of the equations. By the viscosity approach, we show the existence of periodic solutions of the problem under consideration. More precisely, with the help of the subdifferential operator theory and Leray–Schauder theorem, we show the existence of periodic solutions to the approximation problem and then obtain the solution of the original problem by using a passage-to-limit procedure.

1. Introduction

Pattern formation is a widely occurring process in biological systems, which plays a key role in development of organisms. To understand the principles underlying these processes, mathematical modeling is crucial and may yield important insights for identifying relevant mechanisms [1,2]. It is observed that the mechanisms of pattern formation in biology are extremely diverse and in particular conclude Turing instability, which can provide the initial impetus for other mechanisms of pattern formation that dominate the later stages of patterning and be realized in various types of reaction–diffusion systems such as the prey–predator system with a fast-diffusing predator [3–5]. It is also noted that in some situations hysteresis-driven is an important mechanism of pattern formation like fairy rings or periodic bands [6–8]. Actually, Hoppensteadt and Jäger [7] and Hoppensteadt et al. [9] used a hysteresis model to explain that ring formation, in which the growth rate of cells was assumed to has a hysteresis dependence upon nutrient and buffer.

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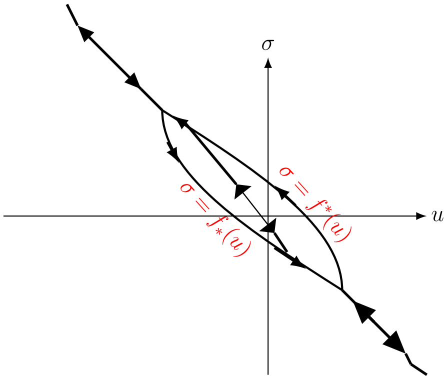

Fig. 1. u is an input, while σ is the corresponding output.

In this paper, we are concerned with the following hysteretic reaction–diffusion system

$$\begin{cases} \sigma_t + u_t + \partial I_u(\sigma) \ni f(x,t,\sigma,u) & \text{in } Q := \Omega \times (0,+\infty), \\ u_t - \triangle u = u(\sigma - u) & \text{in } Q, \end{cases} \tag{1.1}$$

subject to Neumann boundary condition

$$\frac{\partial u}{\partial \nu} = 0 \quad \text{on } \Gamma := \partial\Omega \times (0,+\infty) \tag{1.2}$$

and periodic conditions

$$u(x,t) = u(x,t+\omega), \qquad \sigma(x,t) = \sigma(x,t+\omega) \quad \text{in } \Omega. \tag{1.3}$$

Here Ω is a bounded domain in $\mathbb{R}^N (N \geq 1)$ with smooth boundary $\partial\Omega$, $\triangle = \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2}$, $\frac{\partial}{\partial \nu}$ denotes the outward normal derivative on $\partial\Omega$, ω is a positive constant and f is the given function specified below. $\partial I_u(\sigma)$ is the subdifferential of the indicator function $I_u(\sigma)$, namely,

$$\partial I_u(\sigma) = \begin{cases} \emptyset & \text{if } \sigma > f^*(u) \text{ or } \sigma < f_*(u), \\ [0,+\infty) & \text{if } \sigma = f^*(u) > f_*(u), \\ \{0\} & \text{if } f_*(u) < \sigma < f^*(u), \\ (-\infty, 0] & \text{if } \sigma = f_*(u) < f^*(u), \\ \mathbb{R} & \text{if } \sigma = f^*(u) = f_*(u), \end{cases} \tag{1.4}$$

where f_*, f^* are the non-increasing and continuous functions and will be prescribed further below.

Hysteresis phenomena occur in many fields such as phase transitions, superconductivity and biology [10–14]. According to [14], the hysteresis is referred to as a rate independent memory effect and hysteresis operator becomes an important concept to analyze the mathematical properties of some hysteresis models (see [11,15–17]). As pointed out by Visintin in [14] that some kinds of hysteresis operators can be represented by ordinary differential inclusion containing subdifferential of the indicator function of a closed interval. Indeed, the function σ in the first equation of (1.1) with $f = 0$ is determined by the so called stop operator illustrated in Fig. 1, where f^* and f_* are upper and lower curves, respectively (see T. Aiki, E. Minchev [18], Visintin [14] and Brokate and Sprekels [11]).

The reaction–diffusion equations with hysteresis have been considerably studied during the last decades (see [18,19,11,20–22,14,23] and references therein). In particular, some research have been done on the hysteretic reaction–diffusion equations coming from biochemistry, see Section 5.5 of [5] and [18,19,24,6,25]. It is remarked that P. Gurevich et al. [20,21] studied the reaction–diffusion equations involving a hysteretic discontinuity in the source term, which is defined at each spatial point. They mainly discussed the existence