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Uniform stabilization to equilibrium of a nonlinear fluid–structure interaction model

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ABSTRACT

We consider uniform stability to a *nontrivial* equilibrium of a nonlinear fluid–structure interaction (FSI) defined on a two or three dimensional bounded domain. Stabilization is achieved via boundary and/or interior feedback controls implemented on both the fluid and the structure. The interior damping on the fluid combining with the viscosity effect stabilizes the dynamics of fluid. However, this dissipation propagated from the fluid alone is not sufficient to drive uniformly to equilibrium the entire coupled system. Therefore, additional interior damping on the wave component or boundary porous like damping on the interface is considered. A geometric condition on the interface is needed if only boundary damping on the wave is active. The main technical difficulty is the mismatch of regularity of hyperbolic and parabolic component of the coupled system. This is overcome by considering special multipliers constructed from Stokes solvers. The uniform stabilization result obtained in this article is *global* for the fully coupled FSI model.

1. Introduction

1.1. Overview

In this paper, we study uniform stabilization to a non-trivial equilibria of a fluid–structure interaction model with static interface. There are many physical applications where the equilibria become non-trivial due to applications of external forces. Thus, it is important to be able to control or stabilize the nonlinear model either globally or locally i.e: in a vicinity of such equilibria.

The nonlinear fluid–structure interaction model under consideration is described by the Navier–Stokes equation coupled with wave equation:

$$\begin{cases} y_t - \Delta y + (y\cdot\nabla)y + \nabla p + F_1(y) = f(x) & \text{in } \Omega_f \times (0,T] \equiv Q_f \\ \operatorname{div} y = 0 & \text{in } \Omega_f \times (0,T] \\ w_{tt} = \Delta w + F_2(w) & \text{in } \Omega_s \times (0,T] \equiv Q_s \\ \dfrac{\partial w}{\partial \nu} = \dfrac{\partial y}{\partial \nu} - p\nu + \dfrac{1}{2}(y\cdot\nu)y & \text{on } \Gamma_s \times (0,T] \equiv \Sigma_s \\ y = w_t + F_3(w) & \text{on } \Sigma_s \\ y = 0 & \text{on } \Gamma_f \times (0,T] \equiv \Sigma_f \\ u(0,\cdot) = u_0 & \text{in } \Omega_f \\ w(0,\cdot) = w_0, \qquad w_t(0,\cdot) = w_1 & \text{in } \Omega_s \end{cases} \tag{1}$$

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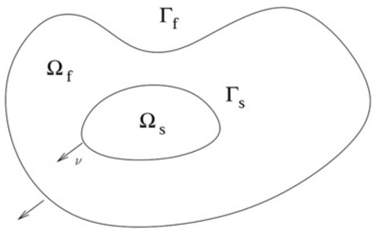

Fig. 1. Geometry of Ω.

where $y(x,t) : Q_f \to \mathbb{R}^d$ is a vector-valued function representing the velocity of the fluid, $p(t,x) : Q_f \to \mathbb{R}$ is a scalar-valued function representing the pressure on Γ_s with respect to the region Ω_s, $w(x,t)$, $w_t(x,t) : Q_s \to \mathbb{R}^d$ are vector-valued functions representing the displacement and velocity of the elastic solid respectively. $f(x)$ is a given force applied to the system. And F_i are feedback mechanism we will identify later.

The physical setting of the model is as follows: Ω_s and Ω_f are two bounded domains in $\mathbb{R}^d$ $(d = 2, 3)$ sharing a common interface Γ_s. ν is the unit outward normal vector on Γ_s with respect to the region Ω_s. The interior domain Ω_s and exterior domain Ω_f are occupied by an elastic solid and incompressible viscous fluid, respectively. The boundary of $\Omega = \Omega_s \cup \Omega_f$ is denoted by Γ_f. See Fig. 1 for the detailed configuration.

System (1) is able to capture small but rapid oscillations of the interaction between the solid and fluid and has numerous applications ranging from naval and aerospace engineering to cell biology and biomedical engineering [2–6]. An important control-theoretic problem is to seek feedback mechanism F_i $(i = 1, 2, 3)$ that could stabilize the interaction near a given equilibrium of system (1).

A triple (y_e, p_e, w_e) with $y_e : \Omega_f \to \mathbb{R}^d$, $p_e : \Omega_f \to \mathbb{R}$, $w_e : \Omega_s \to \mathbb{R}^d$ is an equilibrium solution to the undamped system (1) (with $F_i = 0$) if it satisfies the following stationary problem:

$$\begin{cases} -\Delta y_e + (y_e \cdot \nabla) y_e + \nabla p_e = f & \text{in } \Omega_f \\ \operatorname{div} y_e = 0 & \text{in } \Omega_f \\ \Delta w_e = 0 & \text{in } \Omega_s \\ \dfrac{\partial w_e}{\partial \nu} = \dfrac{\partial y_e}{\partial \nu} - p_e \nu & \text{on } \Gamma_s \\ y_e = 0 & \text{on } \Gamma_f \cup \Gamma_s \end{cases} \tag{2}$$

where $f(x)$ is a given forcing term.

Note that the nonlinear term $\frac{1}{2}(y_e \cdot \nu) y_e$ on Γ_s vanishes since at the equilibrium one has $y_e = 0$ on Γ_s. The interesting case is when the equilibrium is nontrivial—which often results from the effect of the force $f(x)$ applied to the model. In the current paper, the damping is active in the entire domain of Ω_f. A more challenging question to ask is what happens if the damping is active only on a portion of the domain. Such a question can be handled by using technically more involved theory as in [1]. Since in this work our focus is on difficulties brought by the presence of mixed dynamics (hyperbolic/parabolic) and the associated mismatch of regularity between two different types of dynamics, we shall concentrate on full interior fluid stabilization which already represents the main and challenging features of the problem.

1.2. Literature

Fluid–structure interaction has been an active area of research in mathematics, physics and engineering where the interaction between a solid submerged in the fluid or fluid flown in a structure is at the heart of the matter. Examples include cells in the human body fluid or a submarine moving in the water [2–6].

From the mathematical point of view, the major difficulty stems from the mismatch of regularity of boundary traces between the fluid and wave equation. Due to this difficulty, the existence, uniqueness (in three dimensions) of weak solutions of the undamped model in the natural energy level have not been solved until recently, see for example, for linear models [7–9] and for nonlinear models [10,11]). The authors of the former utilized the spectral theory originated in [12] and Lumer-Phillips theorem in conjunction with PDE estimates, while the authors of the latter utilized the hidden regularity of wave equation to overcome this difficulty. In the case of no damping at the interface, but possibly with internal damping in the structure, strong stability to zero equilibrium of the linear model was studied in [7,13] by functional analytic methods such as [12] and others, while polynomial stability was addressed in [14] by energy methods. Both required geometrical conditions with the conclusion being obtained outside the one-dimensional null eigenspace. Finally, in the case of damping at the interface, uniform stability to zero equilibrium without geometric conditions was obtained in [15,16] by using PDE- energy methods (multipliers). A summary of stability results to zero-equilibrium of linear FSI model is available in [17]. Strong and uniform stability to zero equilibrium results have similar counterparts [18–20] for nonlinear 2D models. Regarding stabilization to a nontrivial equilibrium, there are several papers devoted to the topic in the context of Navier–Stokes equation. In that case, one deals with theory of analytic semigroups which described the dynamics of linearization. For example, authors